



**Institute of
Applied Physics**

Friedrich-Schiller-Universität Jena

Imaging and Aberration Theory

Lecture 7: Distortion and coma

2018-11-30

Herbert Gross



Schedule - Imaging and aberration theory 2018

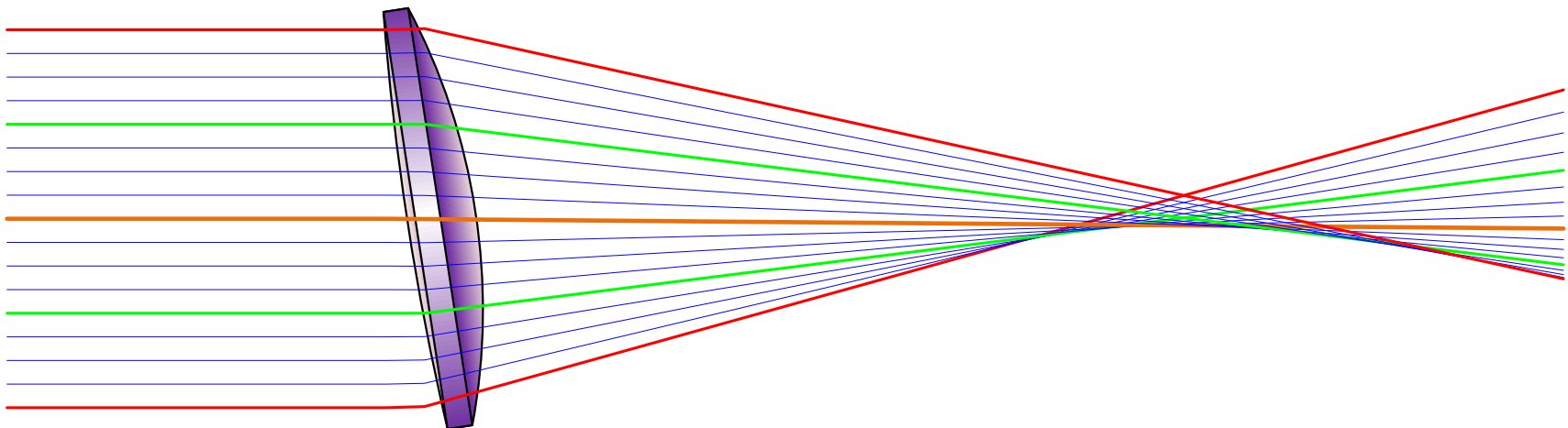
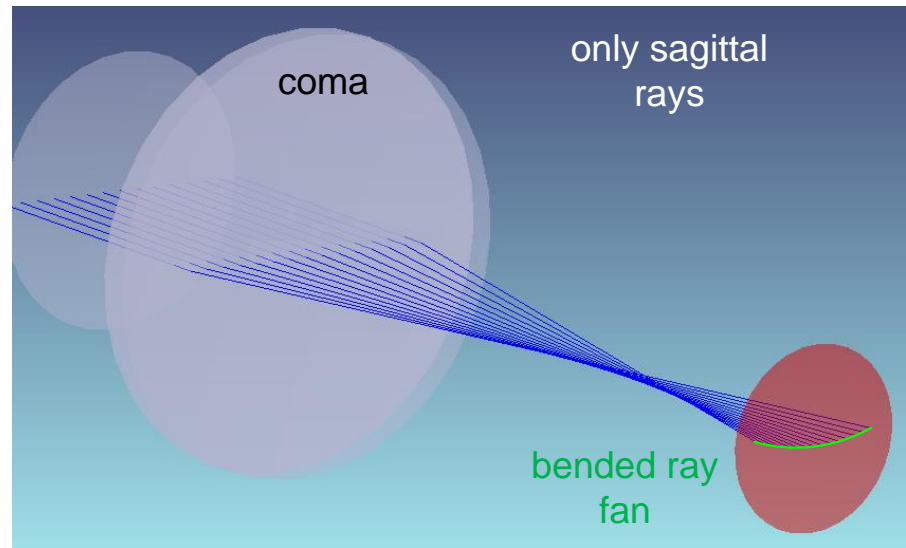
1	19.10.	Paraxial imaging	paraxial optics, fundamental laws of geometrical imaging, compound systems
2	26.10.	Pupils, Fourier optics, Hamiltonian coordinates	pupil definition, basic Fourier relationship, phase space, analogy optics and mechanics, Hamiltonian coordinates
3	02.11.	Eikonal	Fermat principle, stationary phase, Eikonals, relation rays-waves, geometrical approximation, inhomogeneous media
4	09.11.	Aberration expansions	single surface, general Taylor expansion, representations, various orders, stop shift formulas
5	16.11.	Representation of aberrations	different types of representations, fields of application, limitations and pitfalls, measurement of aberrations
6	23.11.	Spherical aberration	phenomenology, sph-free surfaces, skew spherical, correction of sph, aspherical surfaces, higher orders
7	30.11.	Distortion and coma	phenomenology, relation to sine condition, aplanatic systems, effect of stop position, various topics, correction options
8	07.12.	Astigmatism and curvature	phenomenology, Coddington equations, Petzval law, correction options
9	14.12.	Chromatical aberrations	Dispersion, axial chromatical aberration, transverse chromatical aberration, spherochromatism, secondary spectrum
10	21.12.	Sine condition, aplanatism and isoplanatism	Sine condition, isoplanatism, relation to coma and shift invariance, pupil aberrations, Herschel condition, relation to Fourier optics
11	11.01.	Wave aberrations	definition, various expansion forms, propagation of wave aberrations
12	18.01.	Zernike polynomials	special expansion for circular symmetry, problems, calculation, optimal balancing, influence of normalization, measurement
13	25.01.	Point spread function	ideal psf, psf with aberrations, Strehl ratio
14	01.02.	Transfer function	transfer function, resolution and contrast
15	08.02.	Additional topics	Vectorial aberrations, generalized surface contributions, Aldis theorem, intrinsic and induced aberrations, revertability

1. Geometry of coma spot
2. Coma-dependence on lens bending, stop position and spherical aberration
3. Point spread function with coma
4. Distortion
5. Examples

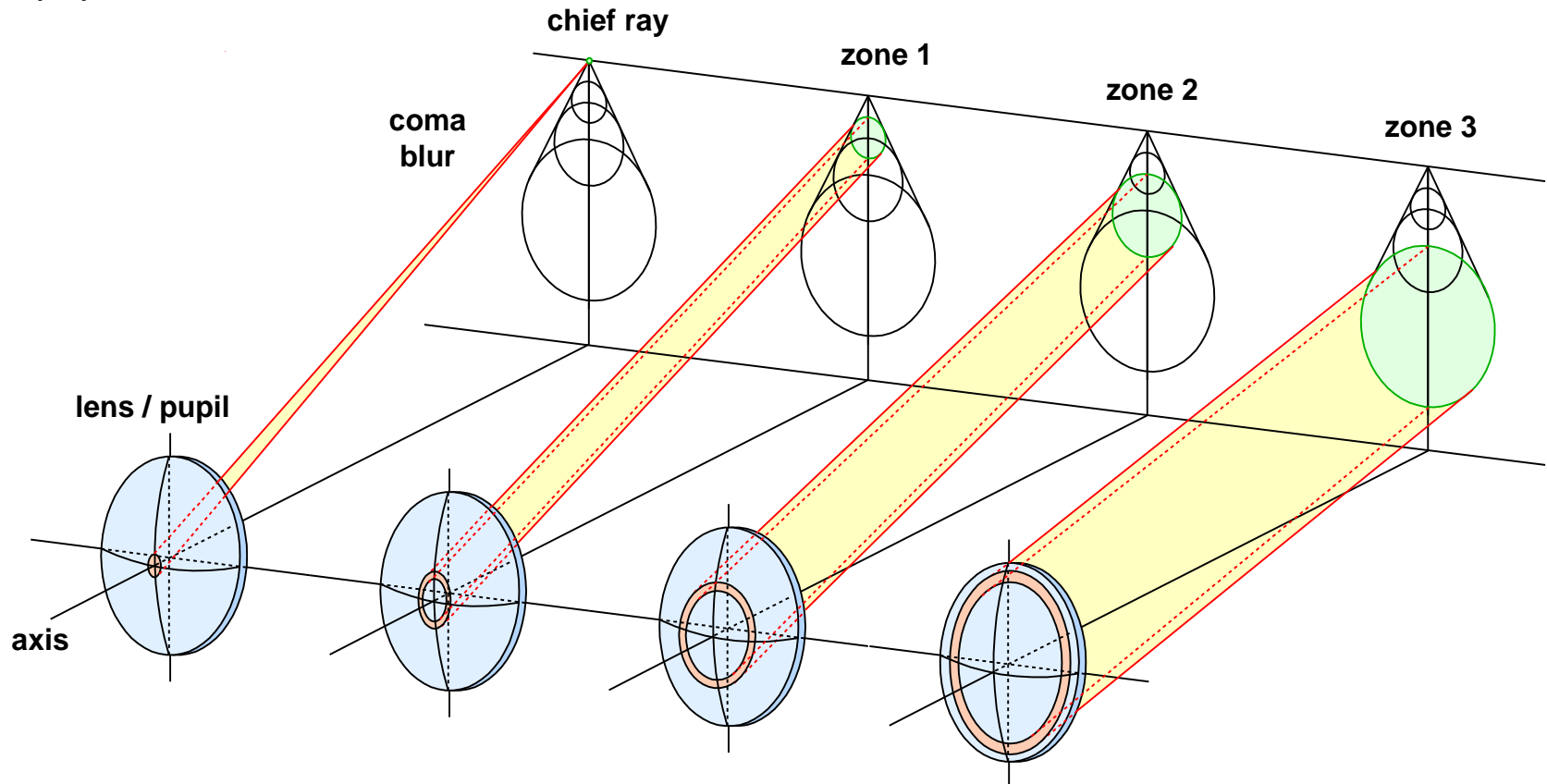


Ray Caustic of Coma

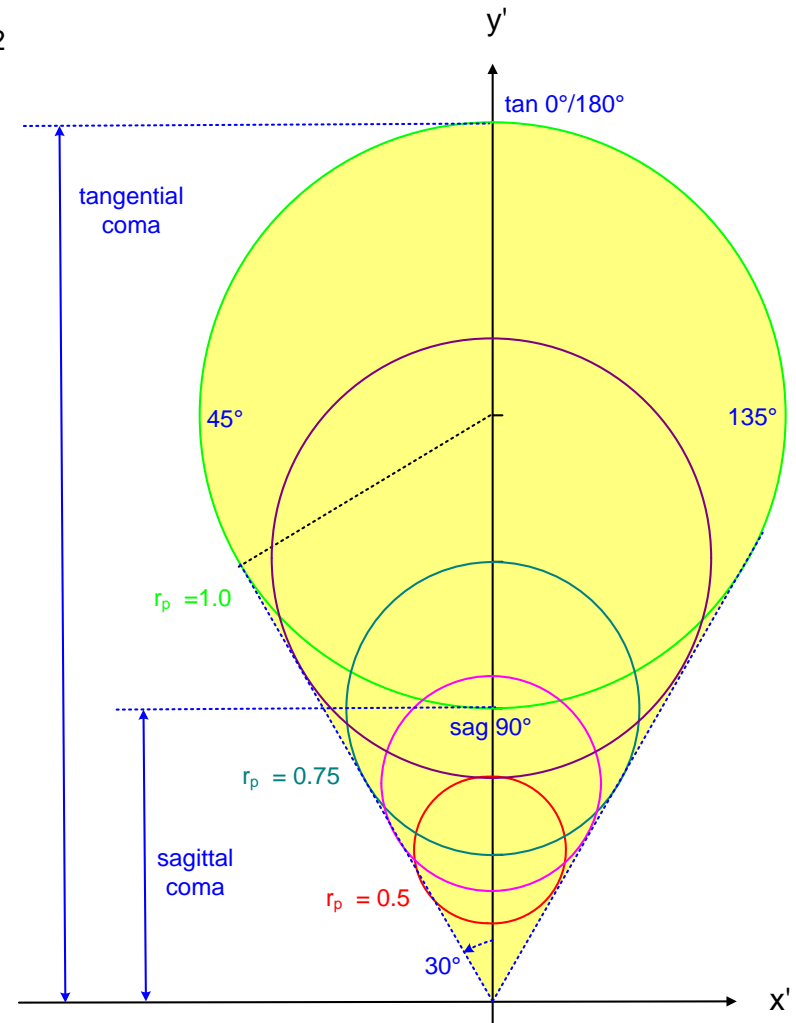
- A sagittal ray fan forms a groove-like surface in the image space
- Tangential ray fan for coma: caustic



- Coma aberration: for oblique bundles and finite aperture due to asymmetry
- Special problem: coma grows linear with field size y
- Systems with large field of view: coma hard to correct
- Relation of spot circles and pupil zones as shown

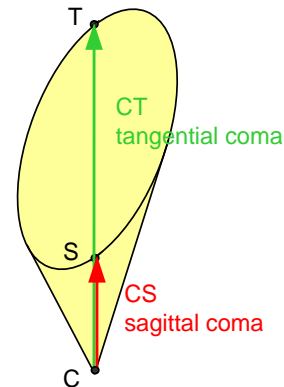
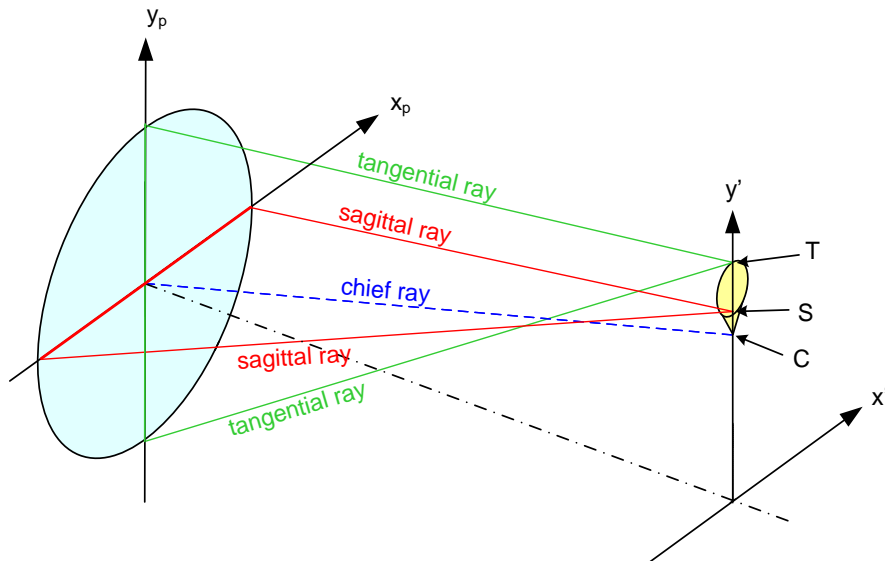
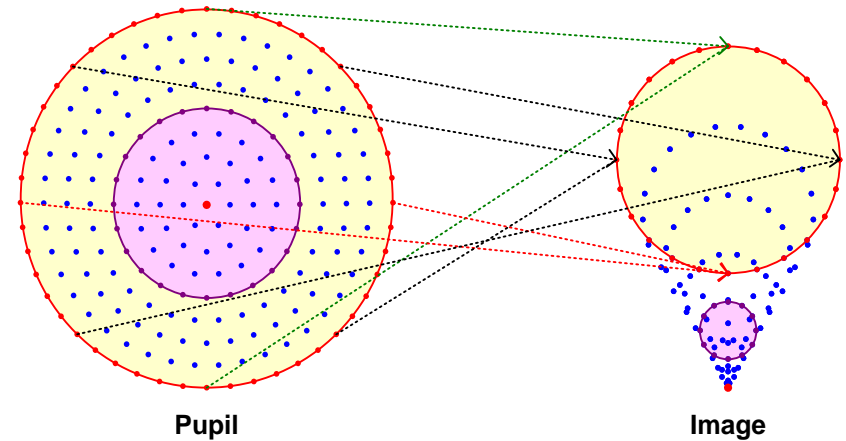


- Coma deviation, elimination of the azimuthal dependence:
circle equation
- Diameter of the circle and position variation with r_p^2
Every zone of the circle generates a circle in the
image plane
- All circles together form a comet-like shape
- The chief ray intersection point is at the tip of
the cone
- The transverse extension of the cone shape has
a ratio of 2:3
the meridional extension is enlarged and gives
a poorer resolution



- Ray trace properties
- Double speed azimuthal growth between pupil and image
- Sagittal coma smaller than tangential coma

$$\Delta y_{\tan} = 3 \cdot \Delta y_{\text{sag}}$$



- Coma Seidel transverse aberrations

$$\Delta y' = S' \cdot r_p'^3 \cos \theta_p + C' \cdot y' \cdot r_p'^2 (2 + \cos 2\theta_p) + (2A' + P') \cdot y'^2 \cdot r_p' \cos \theta_p + D' \cdot y'^3$$

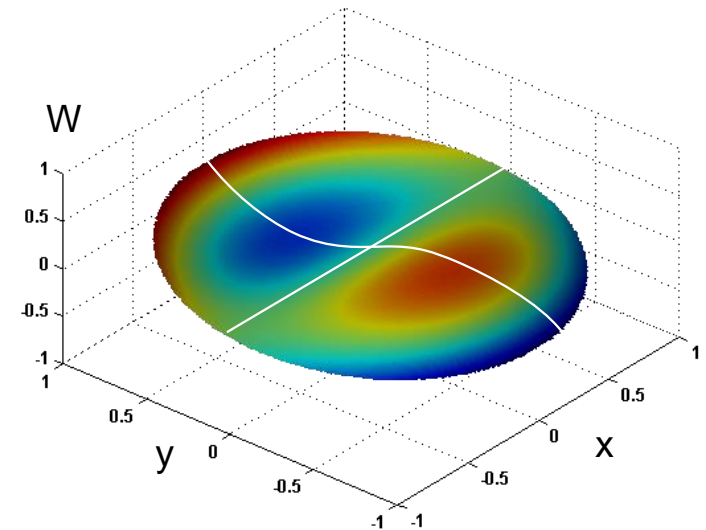
$$\Delta x' = S' \cdot r_p'^3 \sin \theta_p + C' \cdot y' \cdot r_p'^2 \cdot \sin 2\theta_p + P' \cdot y'^2 \cdot r_p' \sin \theta_p$$

- Wavefront for coma

$$W = r_p^3 \cos \theta_p = x_p^2 y_p + y_p^3$$

with

$$x_p = r_p \sin \theta_p, \quad y_p = r_p \cos \theta_p$$



- Relationship

$$\frac{\partial W}{\partial y_p} = -\frac{\Delta y'}{R}, \quad \frac{\partial W}{\partial x_p} = -\frac{\Delta x'}{R}$$

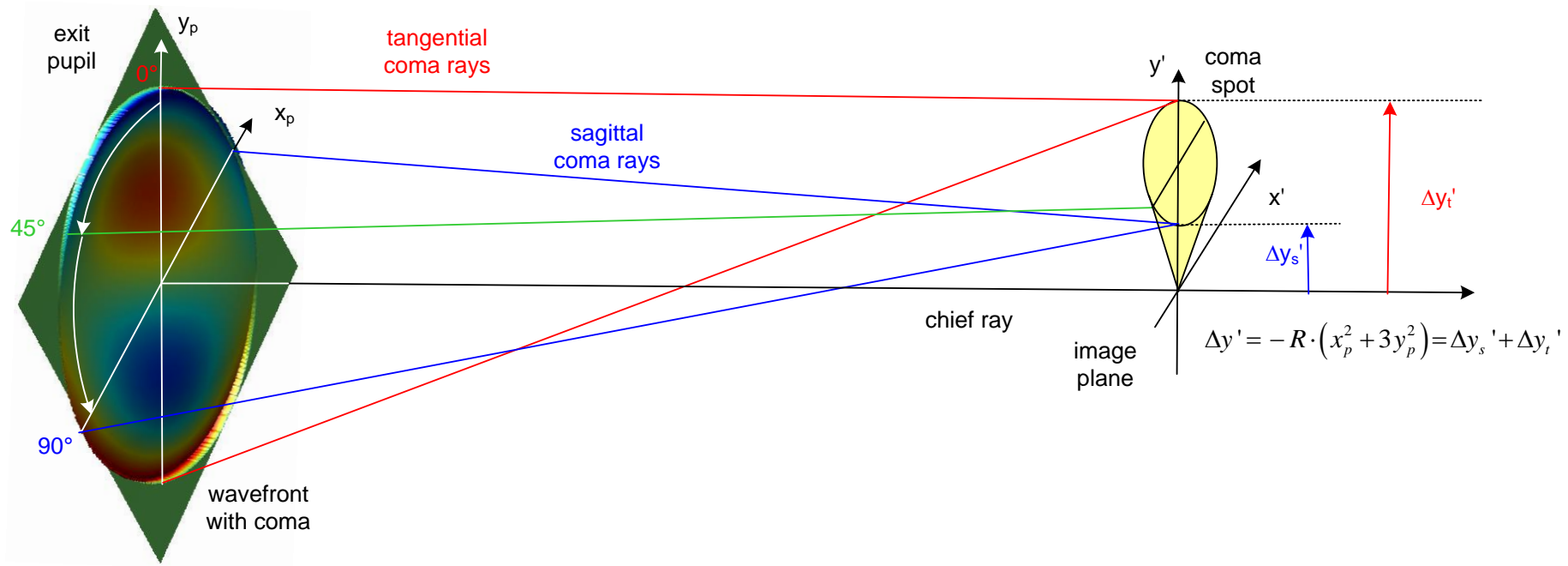
- Here

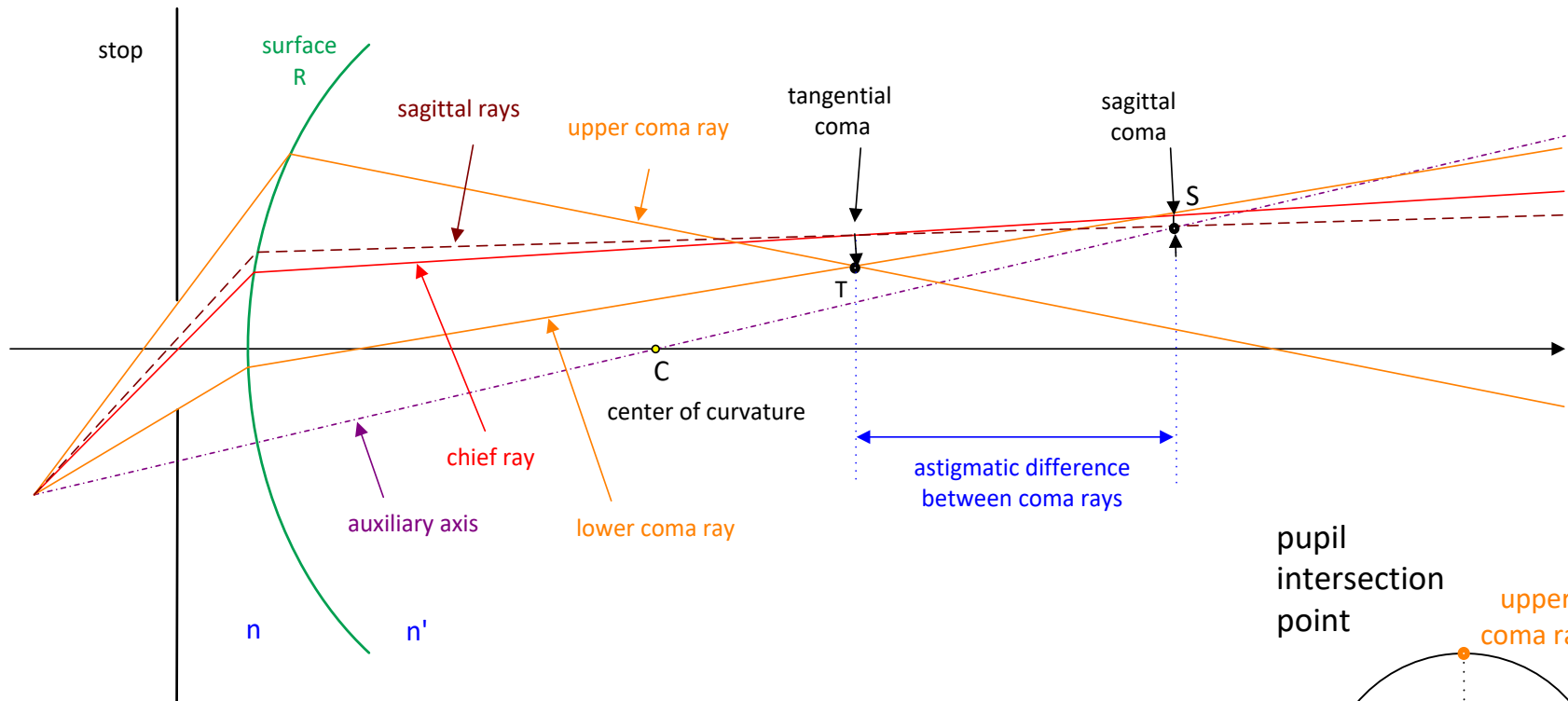
$$\Delta x' = -R \frac{\partial W}{\partial x_p} = -2R \cdot (x_p y_p) = -2R r_p^2 \sin \theta_p \cos \theta_p = -R r_p^2 \sin 2\theta_p$$

$$\Delta y' = -R \frac{\partial W}{\partial y_p} = -R \cdot (x_p^2 + 3y_p^2) = -R r_p^2 (2 + \cos 2\theta_p)$$

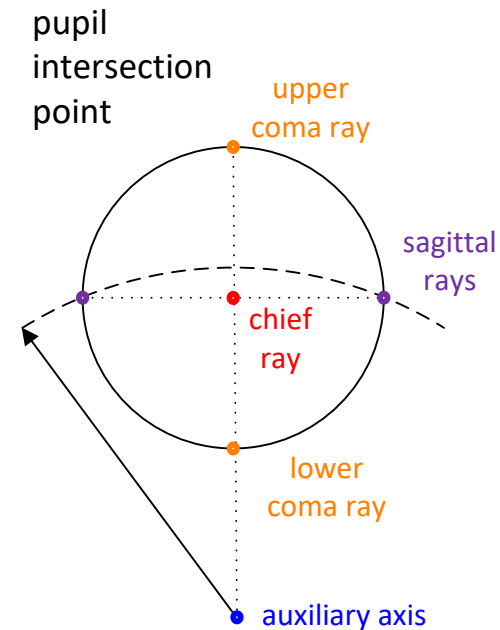
Tangential and Sagittal Coma

- 2 terms of tangential transverse aberration:
 - Sagittal coma depends on x_p , describes the asymmetry
 - Tangential coma depends on y_p , corresponds to spherical aberration under skew conditions larger by a factor of 3
- Only asymmetry removed with sine condition: sagittal coma vanishes

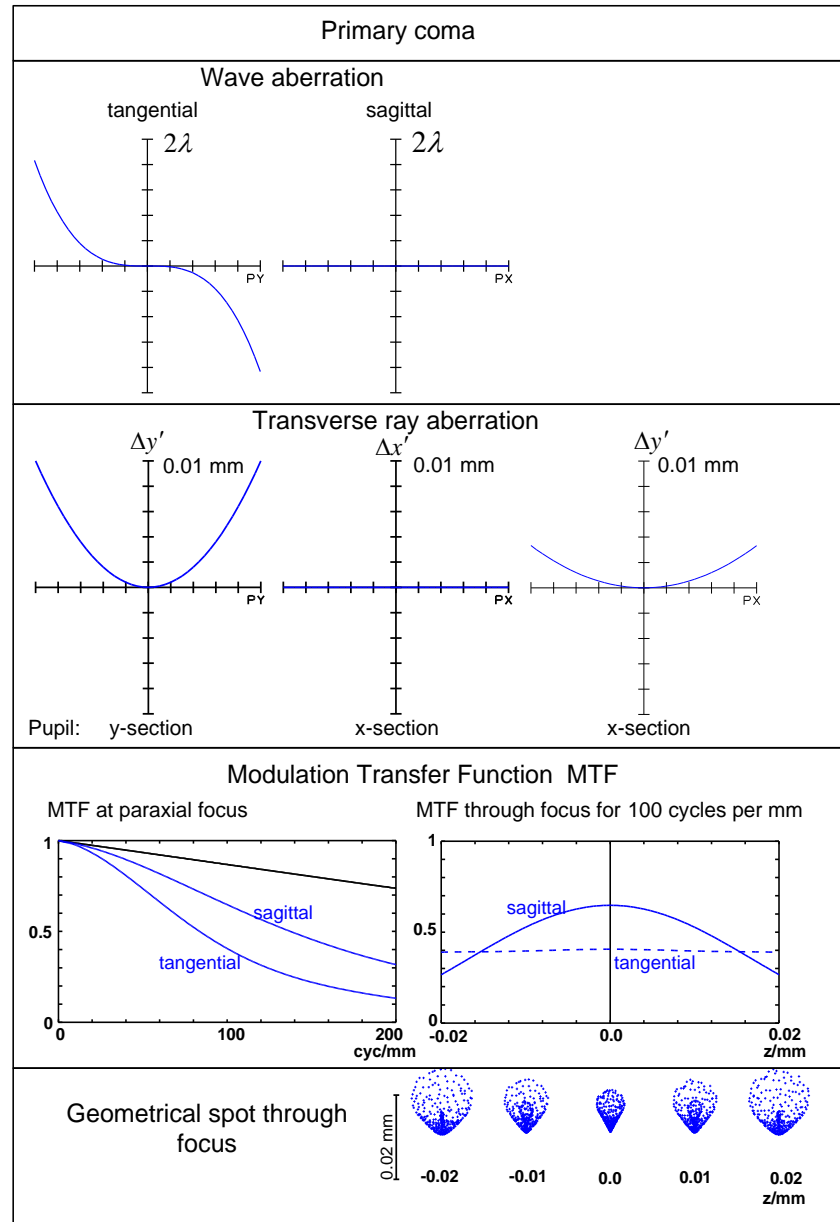




- Occurrence of coma:
skew chief ray and finite aperture
- Asymmetry between upper and lower coma ray
- Bended plane of sagittal coma rays



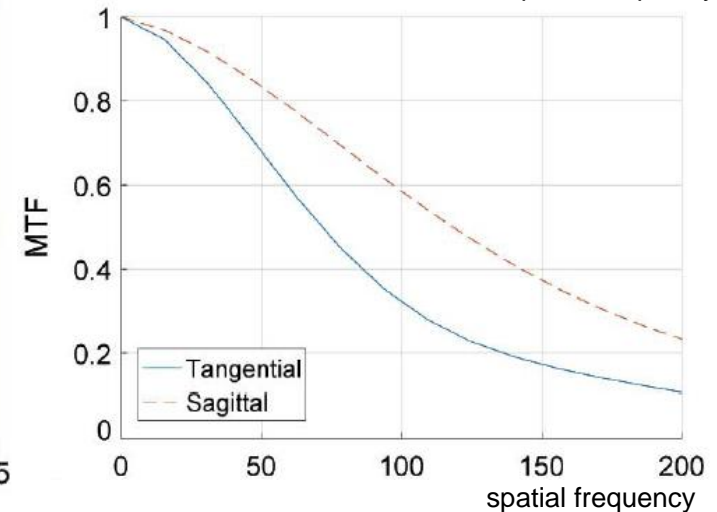
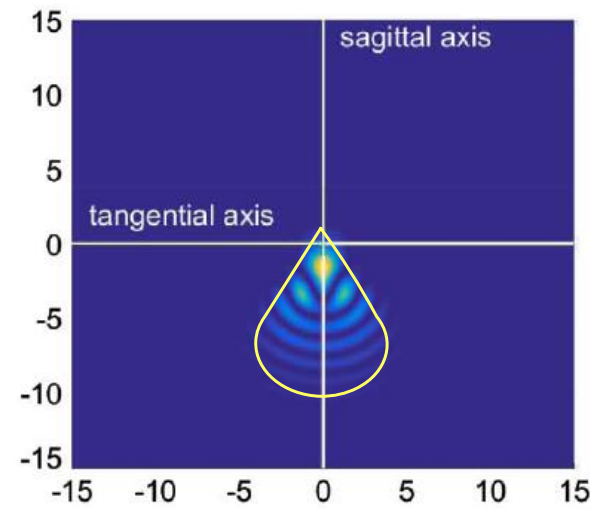
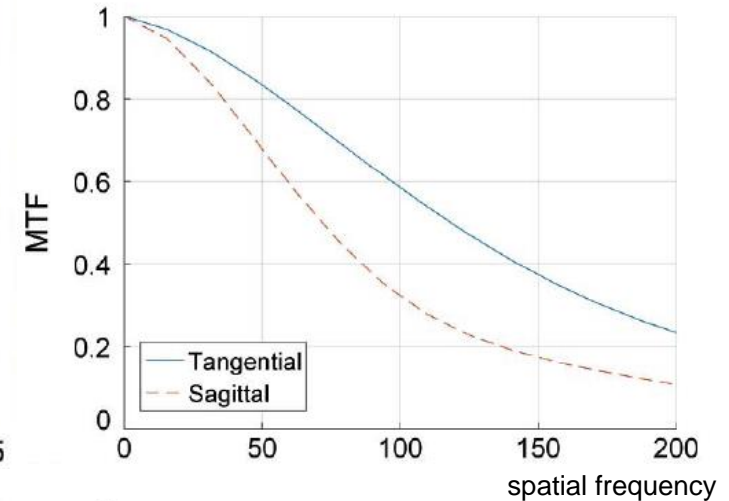
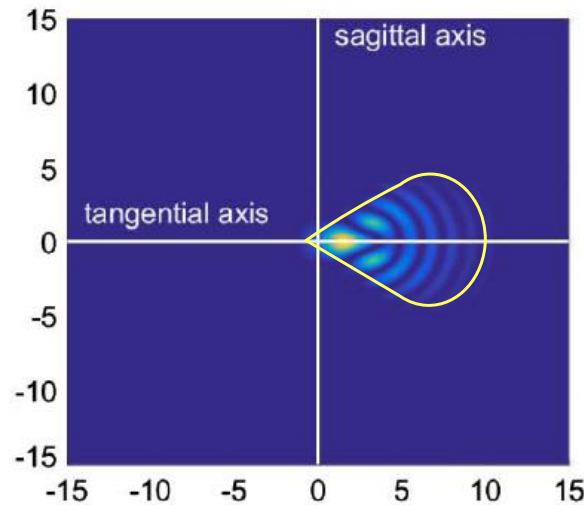
- Typical representations of coma
- Cubic curve in wavefront cross section
- Quadratic function in transverse aberrations





Tangential vs Sagittal Resolution

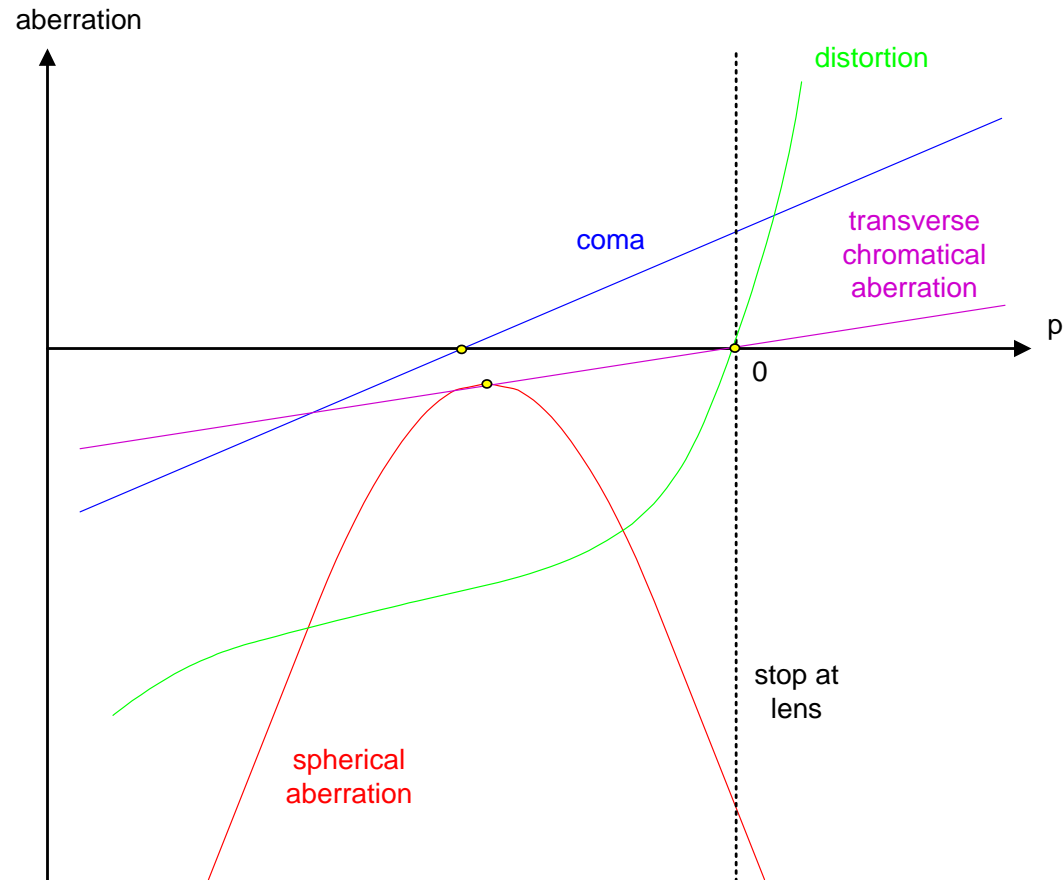
- Asymmetry of the PSF
- Coma 3rd creates a 2:3 diameter pattern
- Usually coma oriented towards the axis, then $MTF_S > MTF_T$ (lower row)



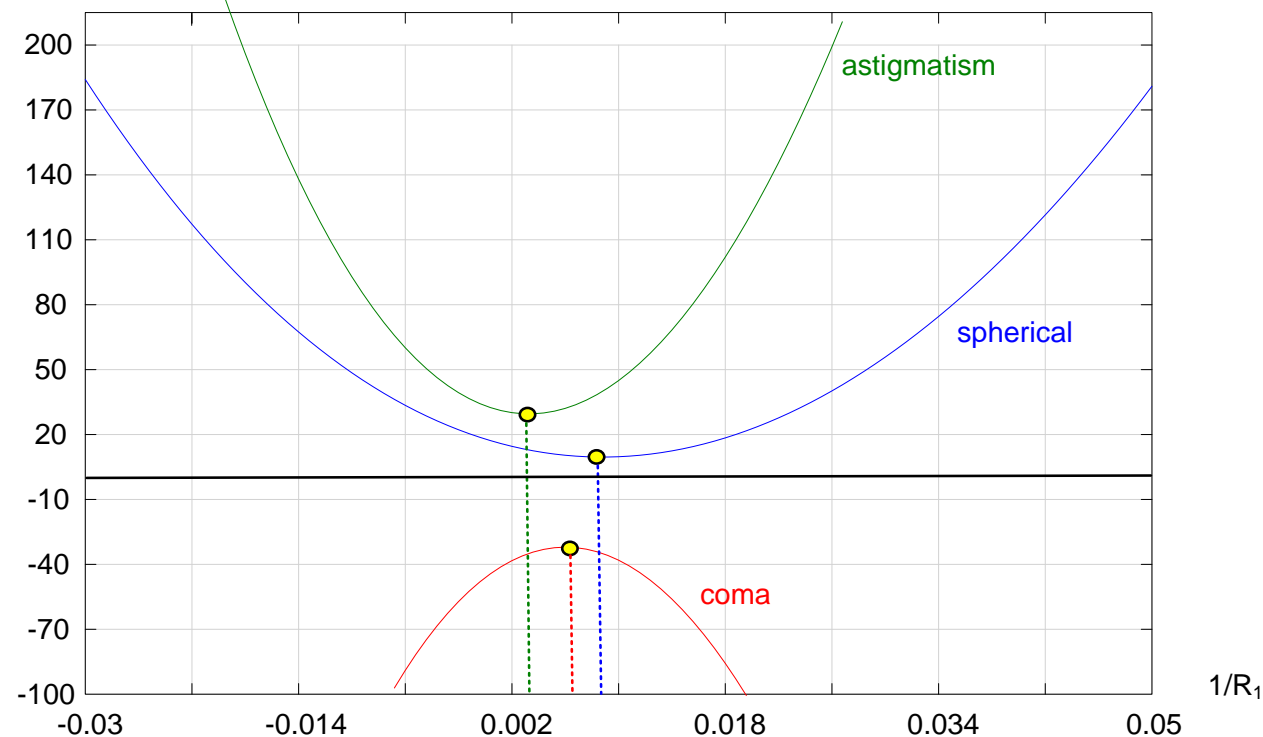


Bending of a Lens

- Bending a single lens with stop at the lens
- Variation of the primary aberrations
- The stop position is important for the off-axis aberrations
- Typical changes:
 1. coma linear
 2. chromatical magnification linear
 3. spherical aberration quadratically



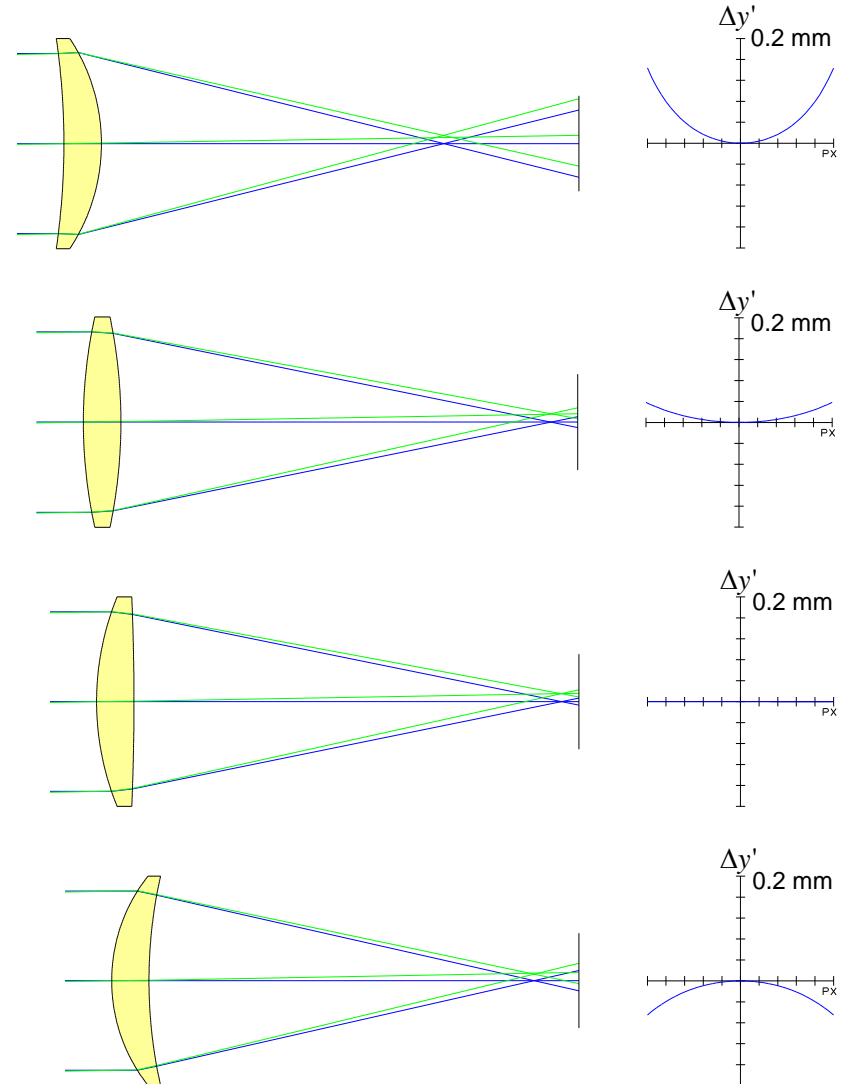
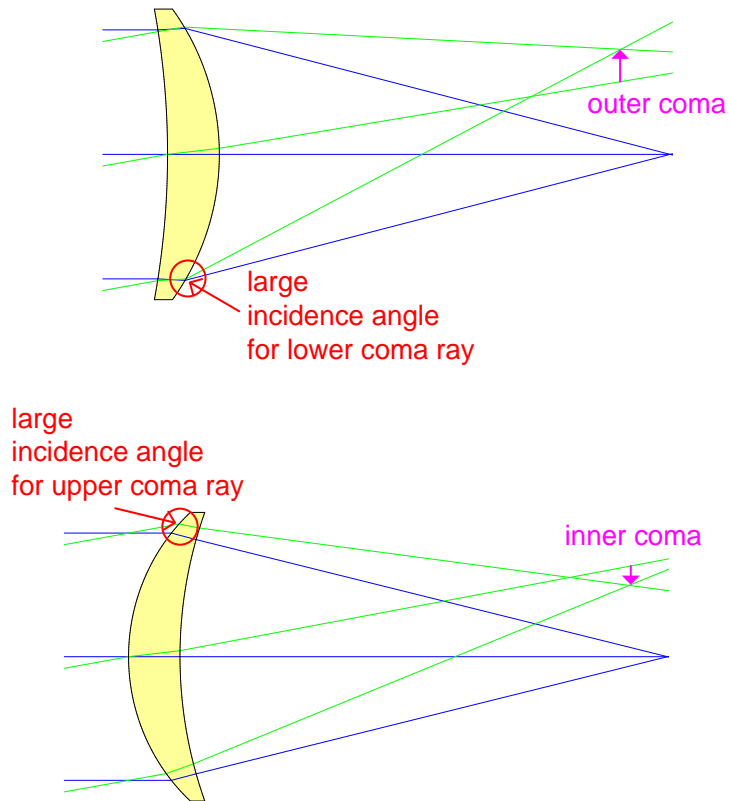
- Lens with remote stop
- Not all of the aberrations spherical, astigmatism and coma can be corrected by bending simultaneously
- Zero correction for coma and astigmatism possible (depends on stop position)
- Spherical aberration not correctable



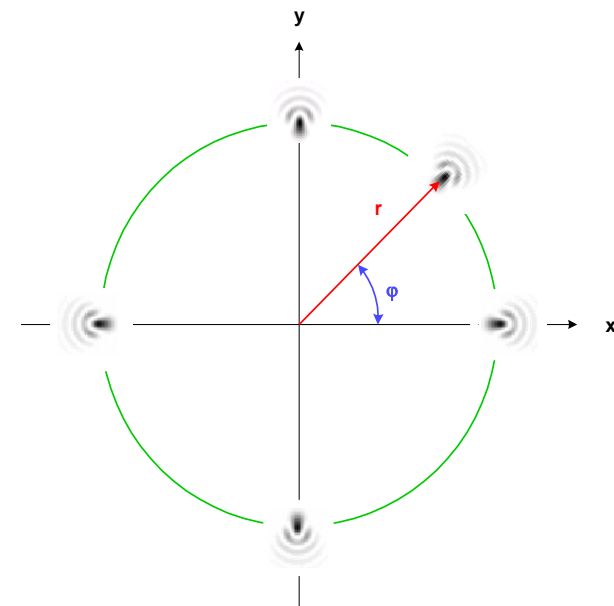
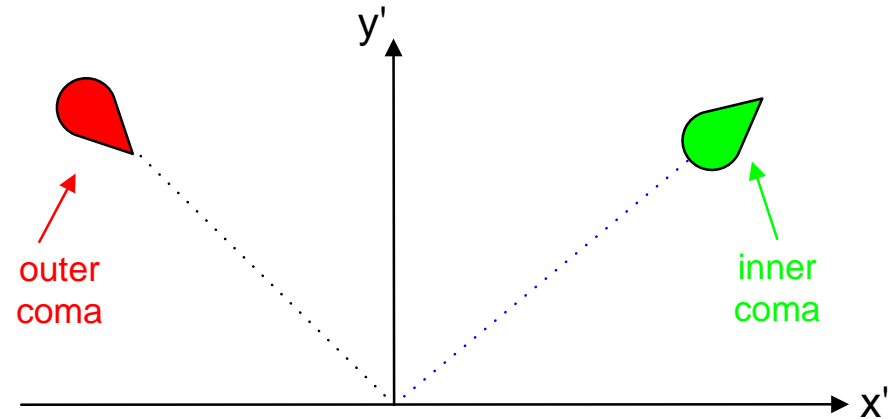


Inner and Outer Coma

- Effect of lens bending on coma
- Sign of coma : inner/outer coma



- Orientation of the coma shape:
distinction between
 1. outer coma, tip towards optical axis
 2. inner coma, tip outside
- Orientation of the coma spot is always rotating with the azimuthal angle of the considered field point





Lens Bending and Natural Stop Position

- The lens contribution of coma is given by if the stop is located at the lens

$$C_{lens} = \frac{1}{4ns'f^2} \cdot \left[\frac{n+1}{n-1} X - (2n+1)M \right]$$

- Therefore the coma can be corrected by bending the lens
- The optimal bending is given by and corrects the 3rd order coma completely

$$X = \frac{(2n+1)(n-1)}{n+1} \cdot M$$

- The stop shift equation for coma is given by with the normalized ratio of the chief ray height to the marginal ray height

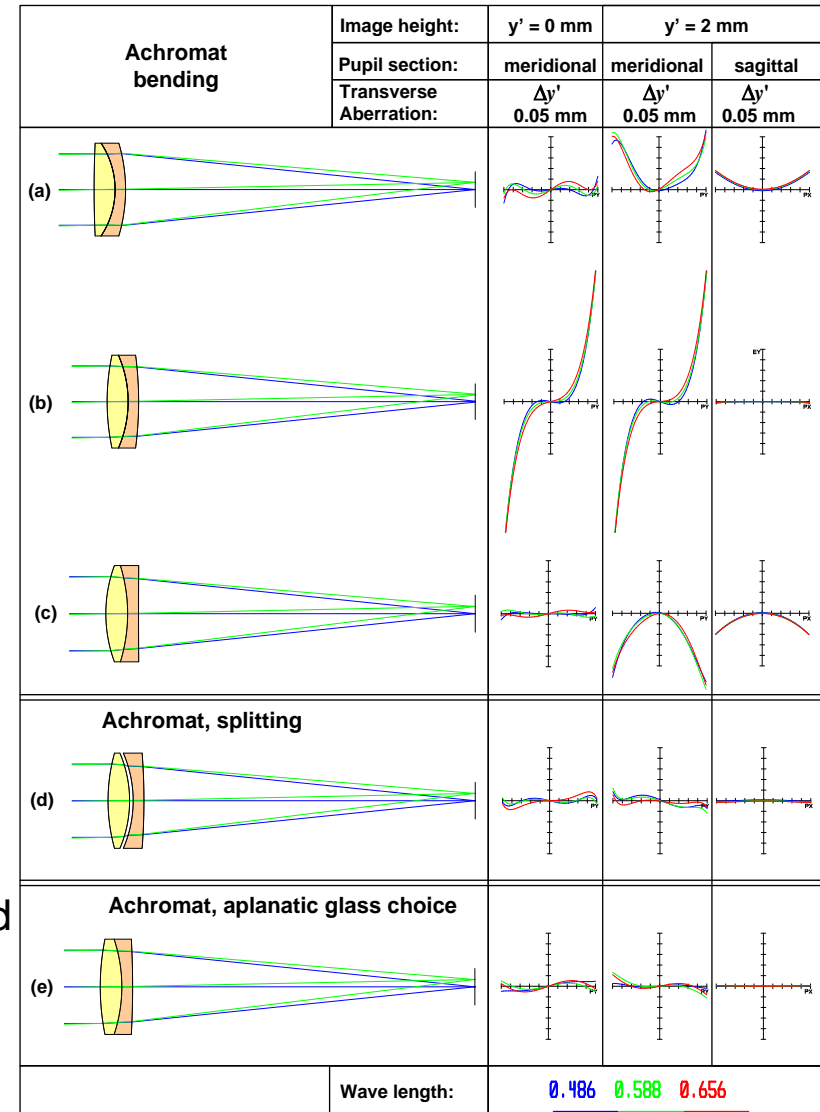
$$S_{II}^* = S_{II} + \delta E \cdot S_I$$

$$\delta E = \frac{\bar{h}_{new} - \bar{h}_{old}}{h}$$

- If the spherical aberration S_I is not corrected, there is a natural stop position with vanishing coma
- If the spherical aberration is corrected (for example by an aspheric surface), the coma doesn't change with the stop position

Coma Correction: Achromate

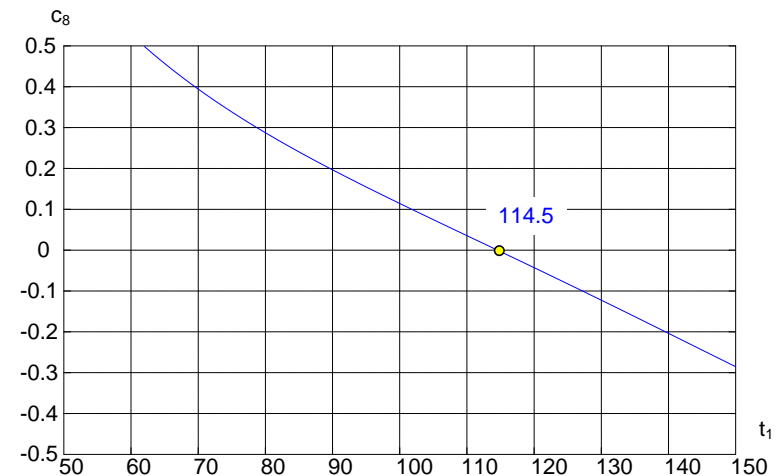
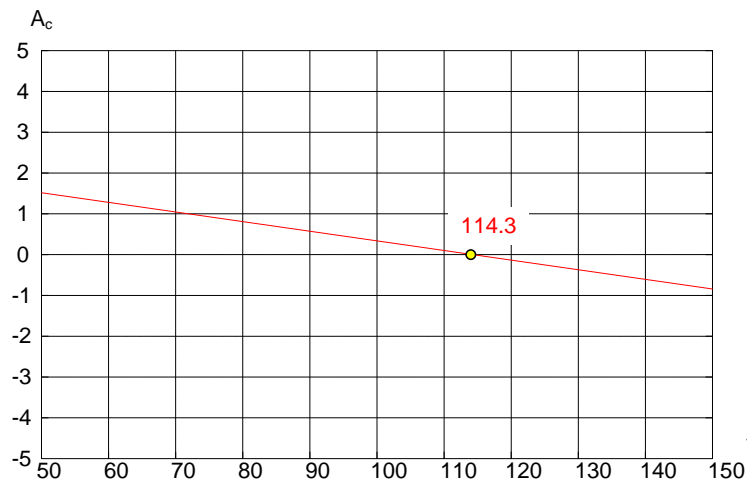
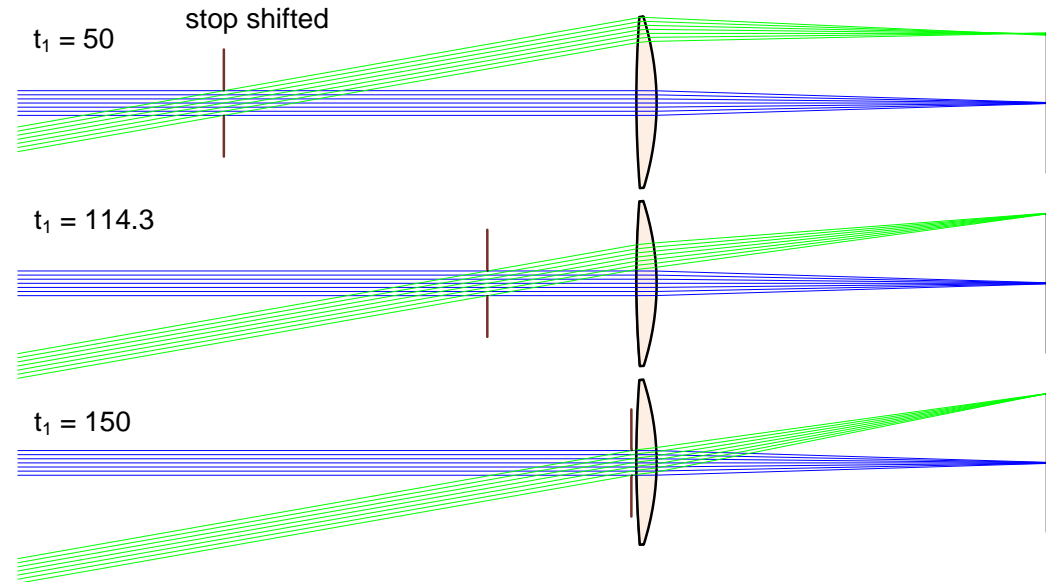
- Bending of an achromate
 - optimal choice: small residual spherical aberration
 - remaining coma for finite field size
- Splitting achromate:
 - additional degree of freedom:
 - better total correction possible
 - high sensitivity of thin air space
- Aplanatic glass choice:
 - vanishing coma
- Cases:
 - a) simple achromate,
 - sph corrected, with coma
 - b) simple achromate,
 - coma corrected by bending, with sph
 - c) other glass choice: sph better, coma reversed
 - d) splitted achromate: all corrected
 - e) aplanatic glass choice: all corrected



Coma-free Stop Position



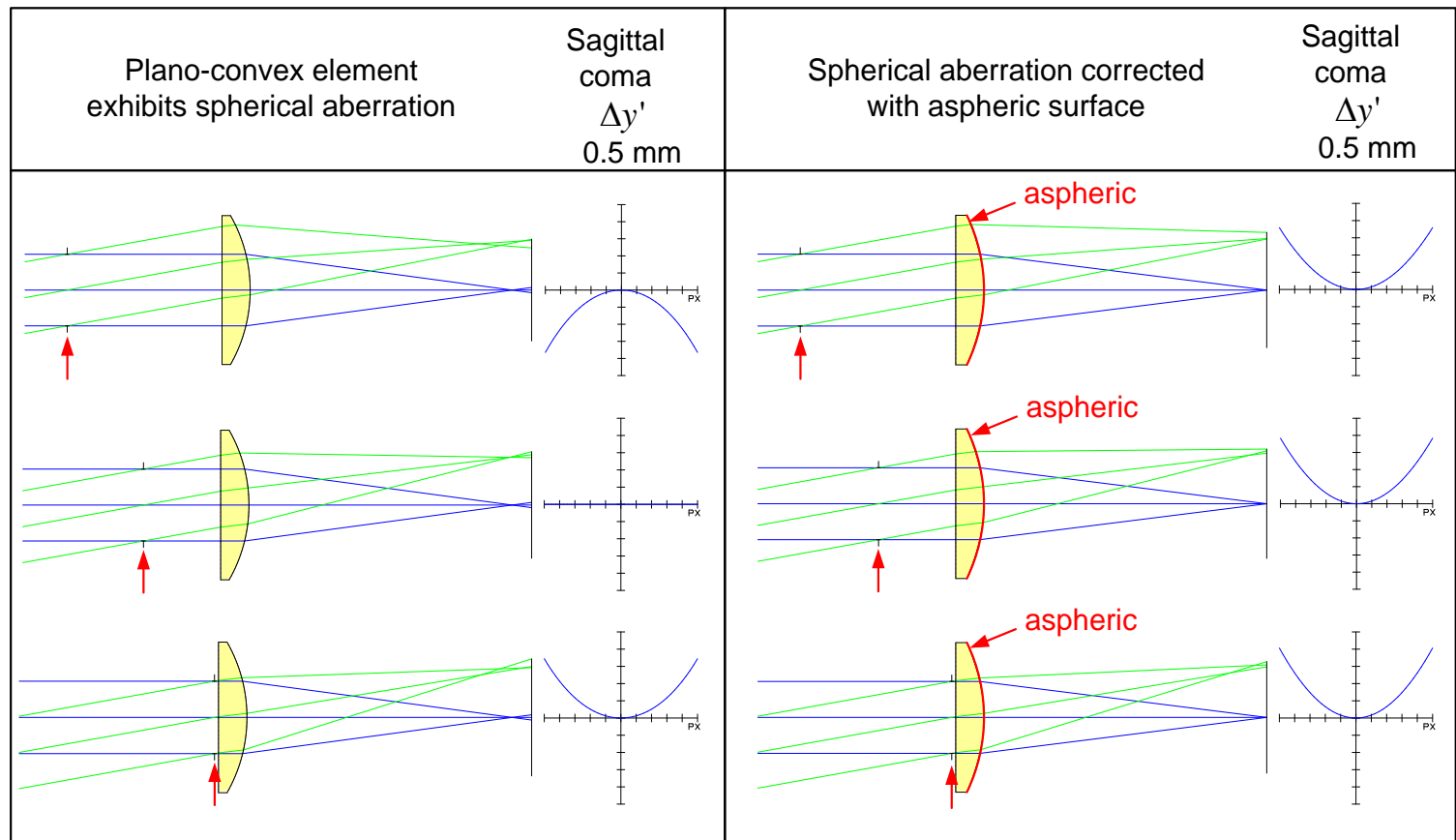
- Example
- The front stop position of a single lens is shifted
- The 3rd order Seidel coefficient as well as the Zernike coefficient vanishes at a certain position of the stop





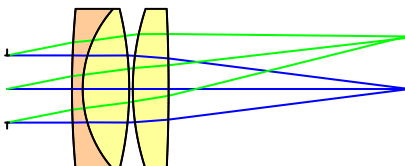
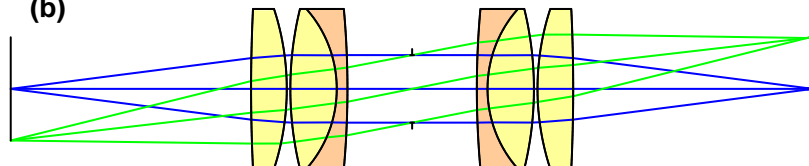
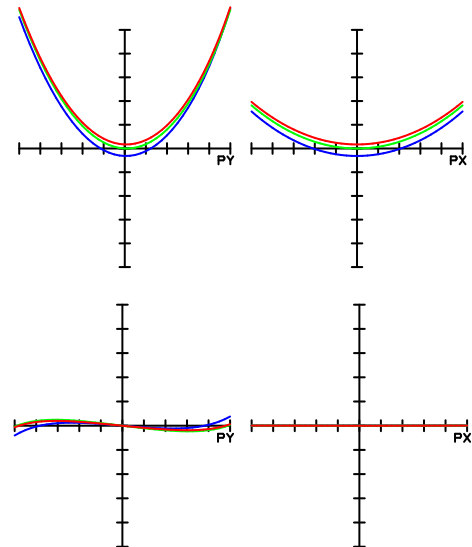
Coma Correction: Stop Position and Aspheres

- Combined effect, aspherical case prevents correction



Coma Correction: Symmetry Principle

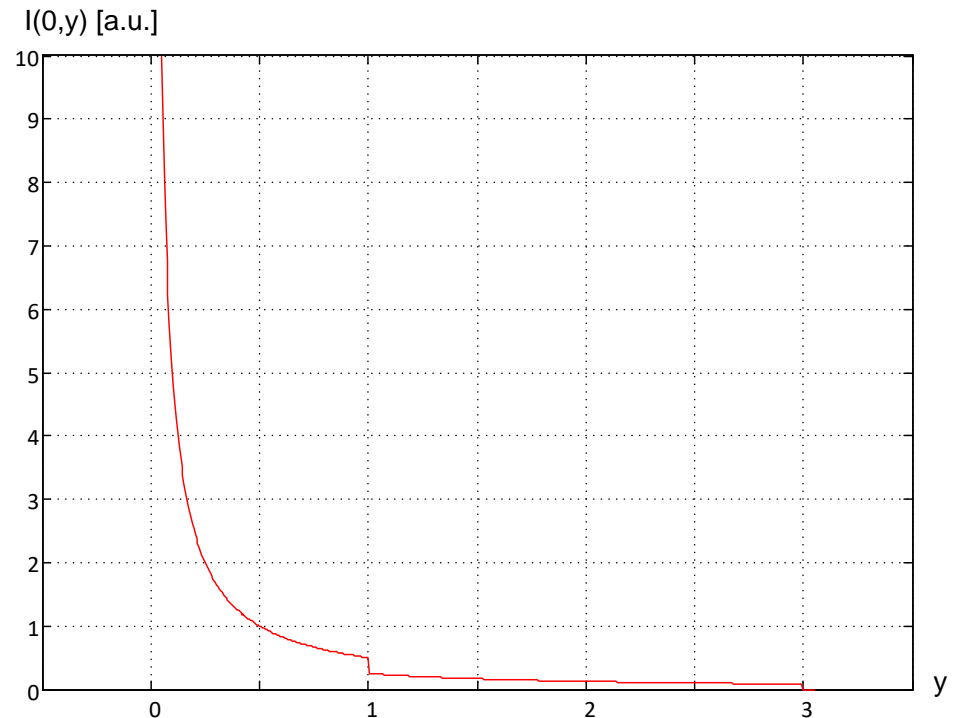
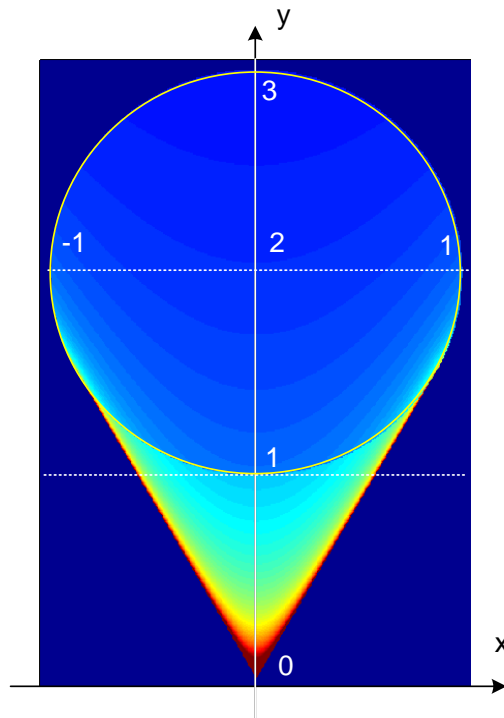
- Perfect coma correction in the case of symmetry
- But magnification $m = -1$ not useful in most practical cases

Symmetry principle	Image height:	$y' = 19 \text{ mm}$	
	Pupil section:	meridional	sagittal
	Transverse Aberration:	$\Delta y'$ 0.5 mm	$\Delta y'$ 0.5 mm
(a)			
(b)			
			

- Geometrical calculated spot intensity
- There is a step at the lower circle boundary
- The peak lies in the apex point
- The centroid lies at the lower circle boundary
- The minimal rms radius is

$$I(x, y) = \begin{cases} \frac{I_{Exp} \cdot a^3}{2A_c \cdot R} \cdot \frac{1}{\sqrt{x^2 - 3y^2}} & \text{inside largest circle} \\ \frac{I_{Exp} \cdot a^3}{A_c \cdot R} \cdot \frac{1}{\sqrt{x^2 - 3y^2}} & \text{else inside coma shape} \end{cases}$$

$$r_{rms} = \sqrt{\frac{2}{3}} \cdot \frac{R \cdot A_c}{a}$$



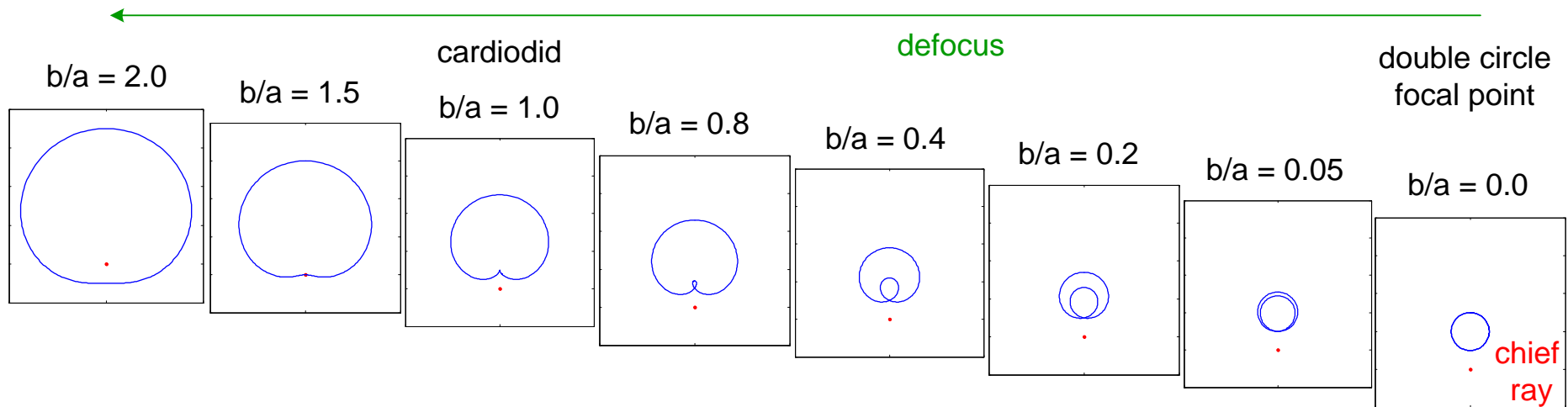


Zonal Curves of Coma with Defocus

- Transverse aberrations in the case of coma and defocus

$$\Delta \vec{H} = -\frac{1}{L} \cdot \left[W_{131} r_p^2 \vec{H} + 2 \cdot (2W_{020} + W_{131} H r_p^2 \cos \theta) \cdot \vec{r}_p \right]$$

- Two deviations:
1st term along field vector
2nd term along pupil vector
- Zonal curve for different defocus values: Limacon of Pascal $H = 2(b + a \cos \theta)$
- special cases cardio / double circle in focal point



PSF for Coma Aberration

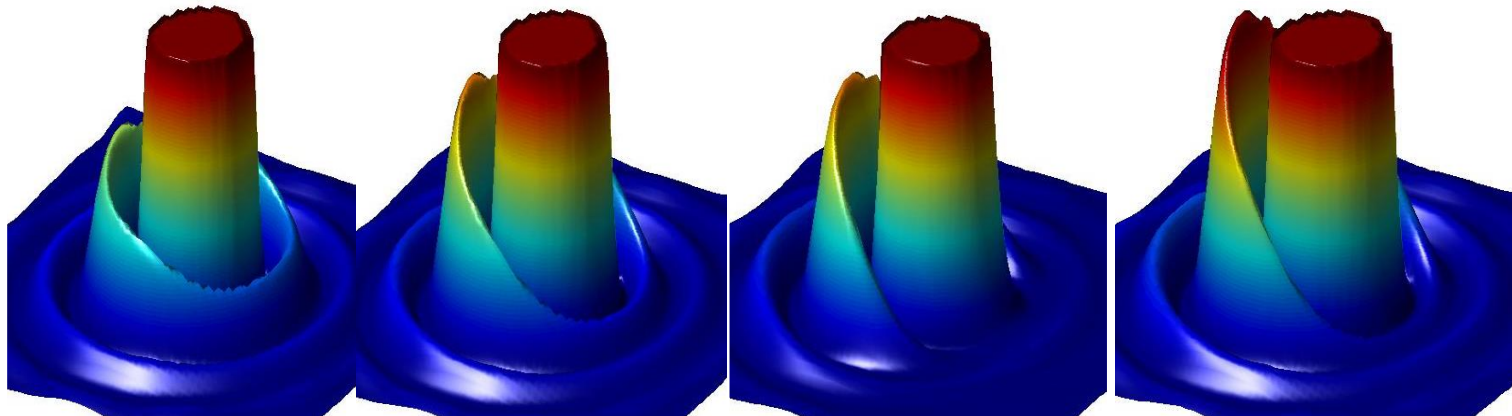
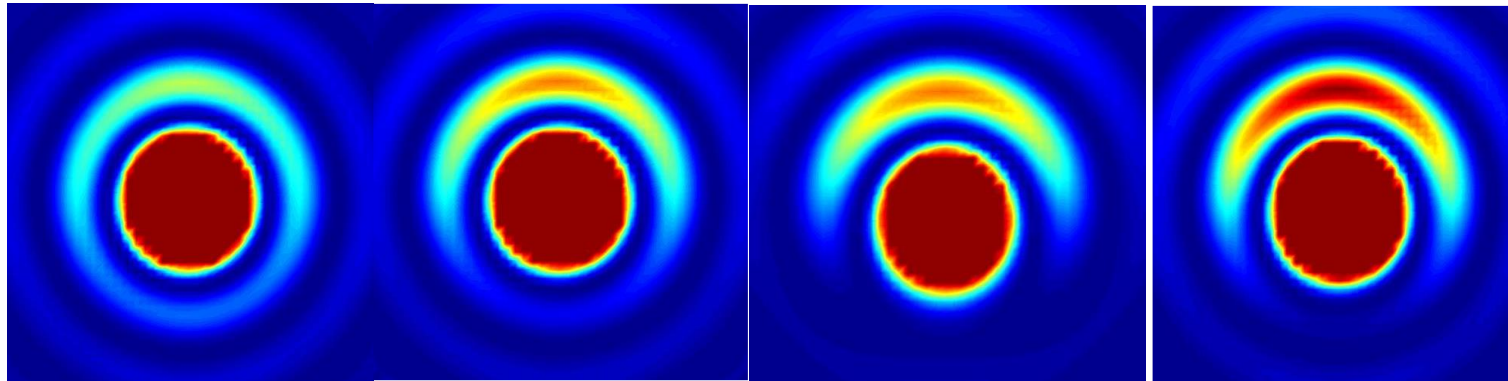
- PSF with coma
- The 1st diffraction ring is influenced very sensitive

$$W_{31} = 0.03 \lambda$$

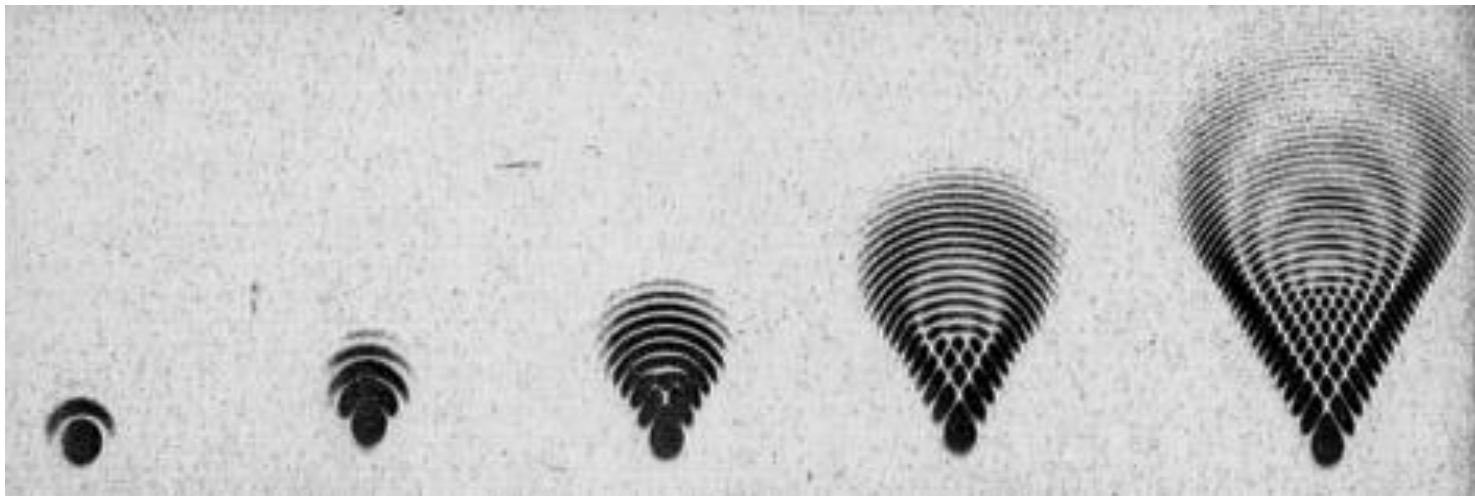
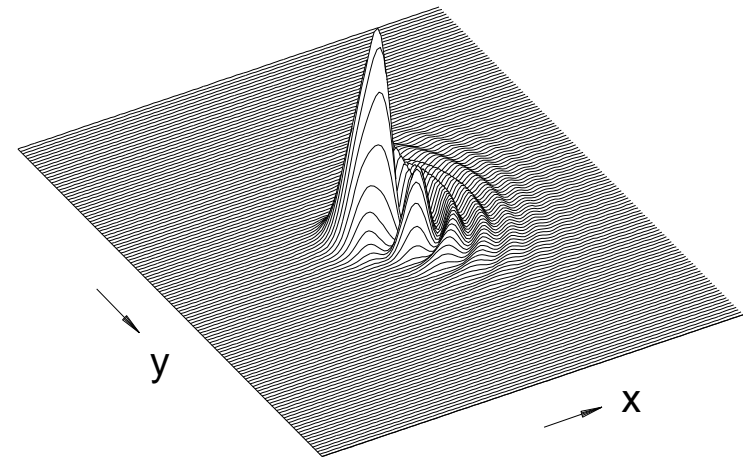
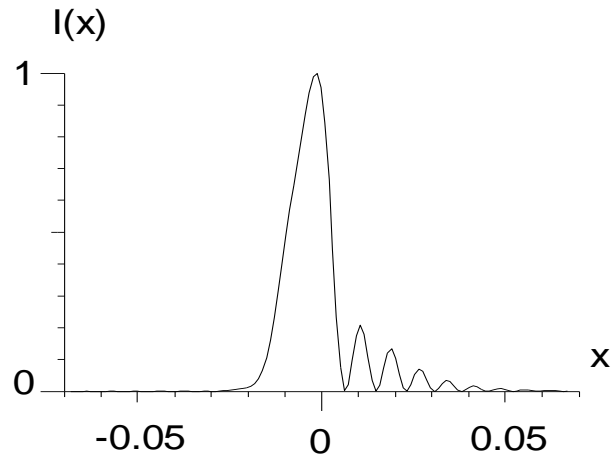
$$W_{31} = 0.06 \lambda$$

$$W_{31} = 0.09 \lambda$$

$$W_{31} = 0.15 \lambda$$



Psf with Coma



$W_{13} = 0.3 \lambda$

$W_{13} = 1.0 \lambda$

$W_{13} = 2.4 \lambda$

$W_{13} = 5.0 \lambda$

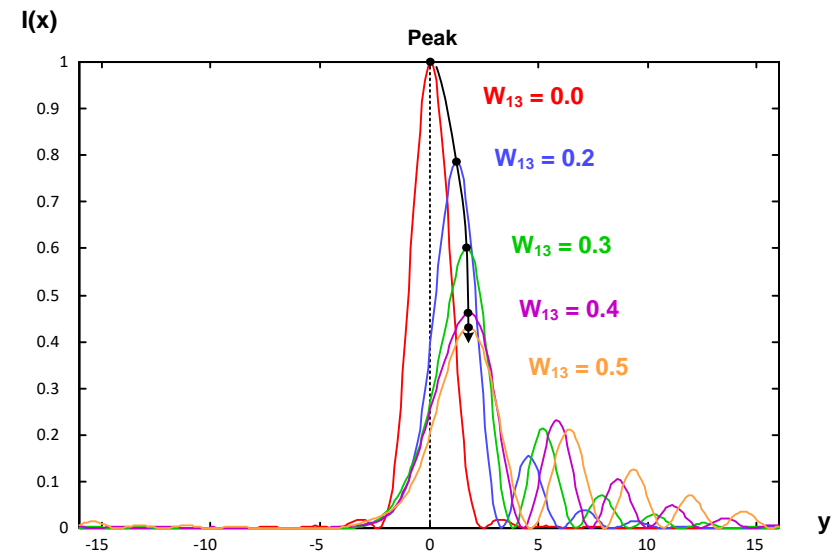
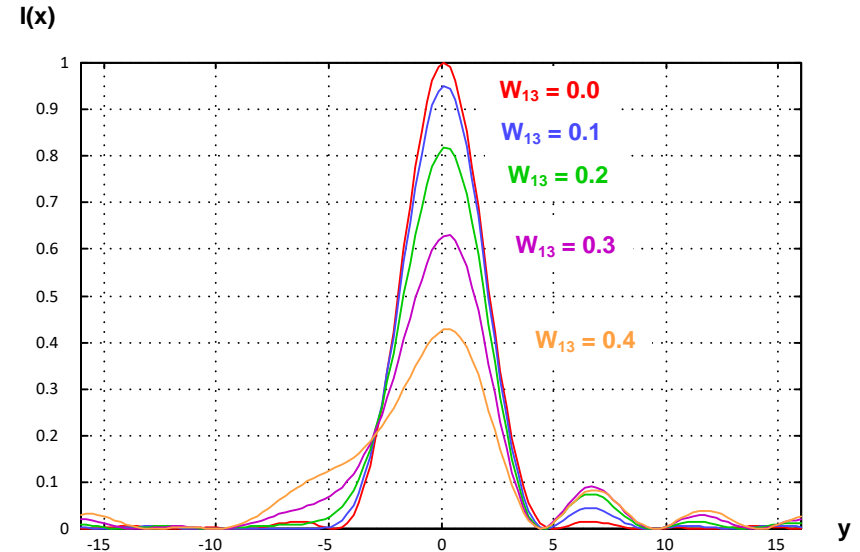
$W_{13} = 10.0 \lambda$



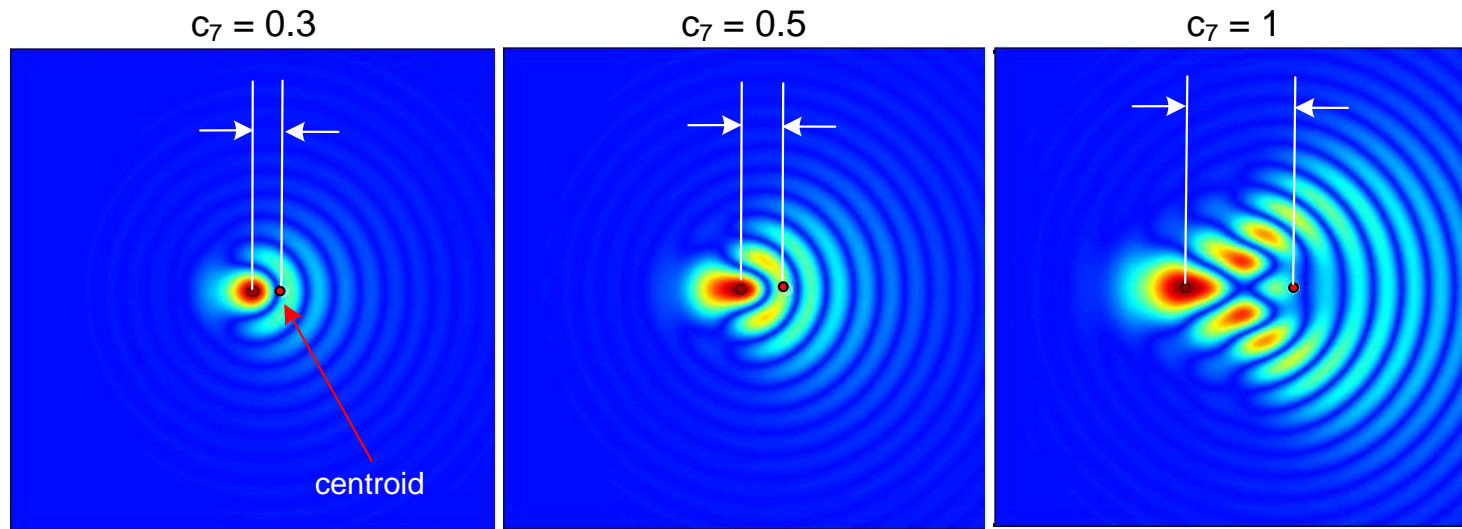
Transversal Psf with Coma

- Change of Zernike coma coefficient
 - peak height reduced
 - peak position constant due to tilt component
 - distribution becomes asymmetrical

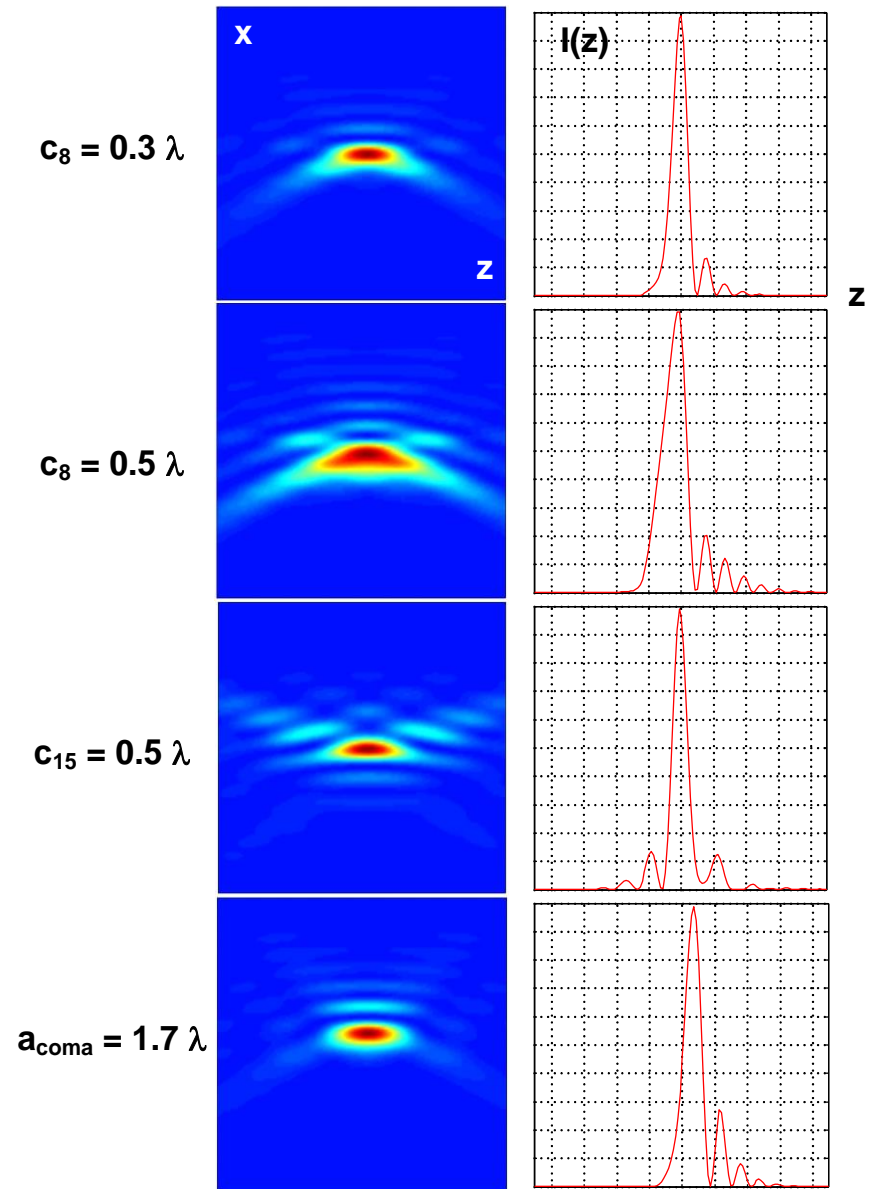
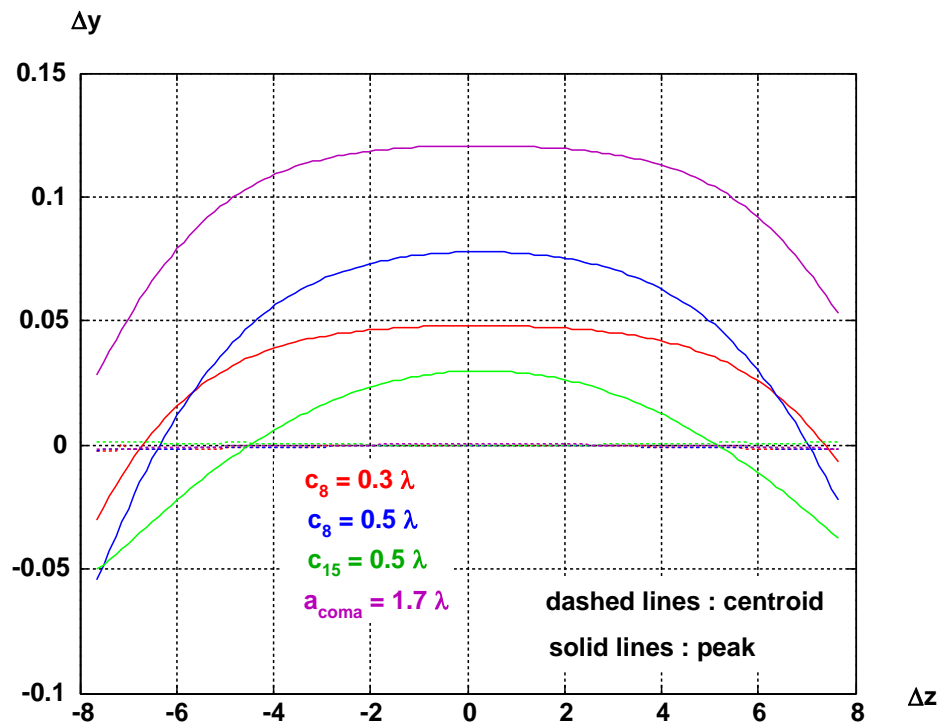
- Change of Seidel coma coefficient
 - peak height reduced
 - peak position moving
 - distribution becomes asymmetrical



- Separation of the peak and the centroid position in a point spread function with coma
- From the energetic point of view coma induces distortion in the image



- Defocus: centroid moves on a straight line (line of sight)
- Peak of intensity moves on a curve (bananicity)



- Centroid of the psf intensity
- Elementary physical argument:
The centroid has to move on a straight line:
line of sight
- Wave aberrations with odd order:
 - centroid shifted
 - peak and centroid are no longer coincident

$$x_s(z) = \frac{\iint x \cdot I(x, y, z) dx dy}{\iint I(x, y, z) dx dy} = \frac{1}{P} \cdot \iint x \cdot I(x, y, z) dx dy$$

$$y_s(z) = \frac{2 \cdot z}{D_{Exp}} \cdot \sum_{n=1,3,5,\dots} \sqrt{2(n+1)} \cdot c_{n1}$$

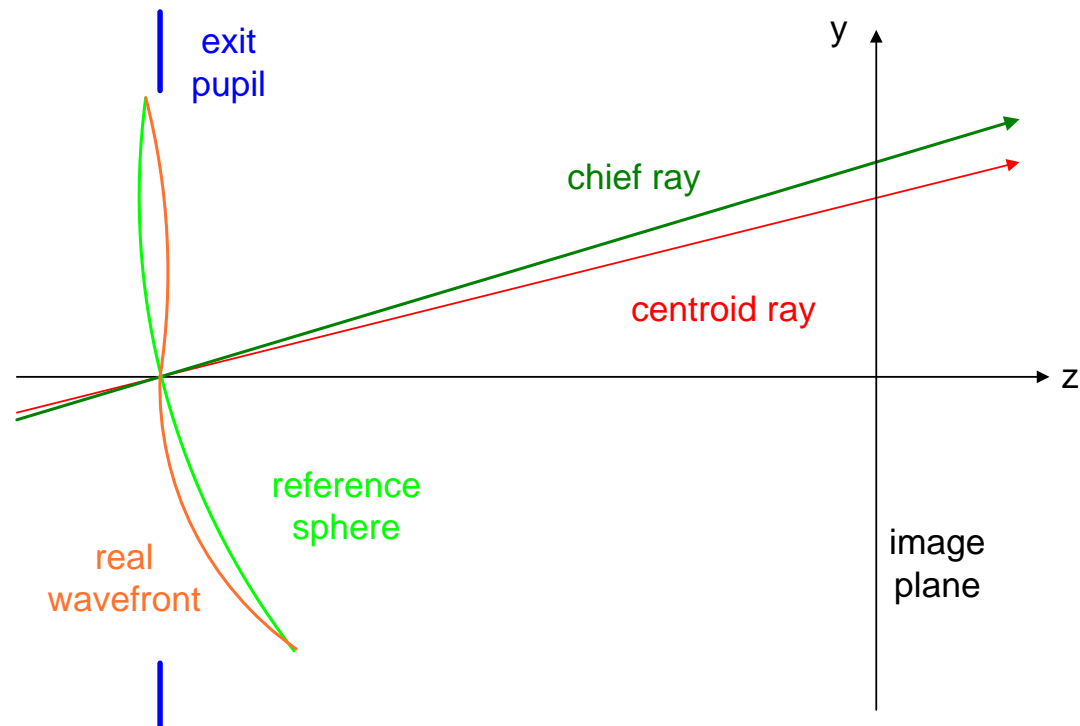
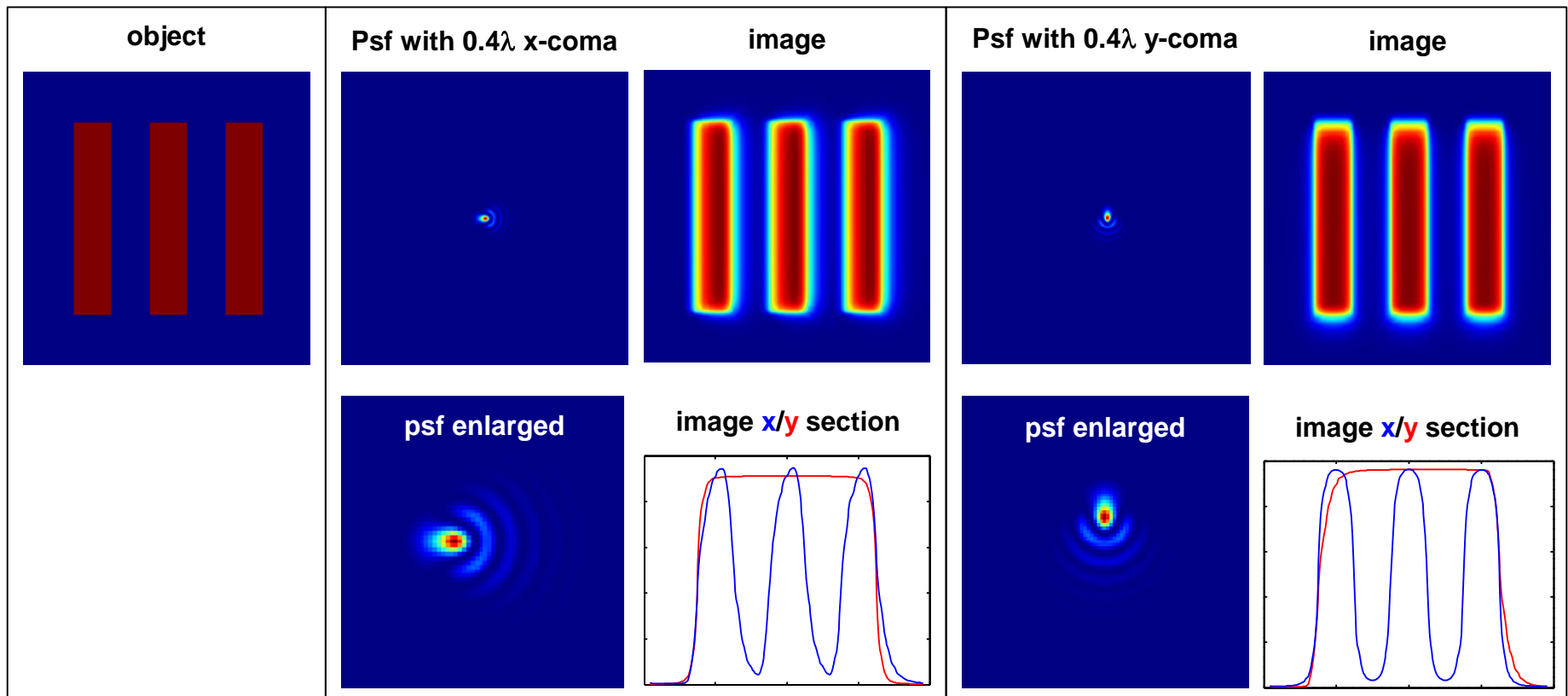


Image Degradation by Coma

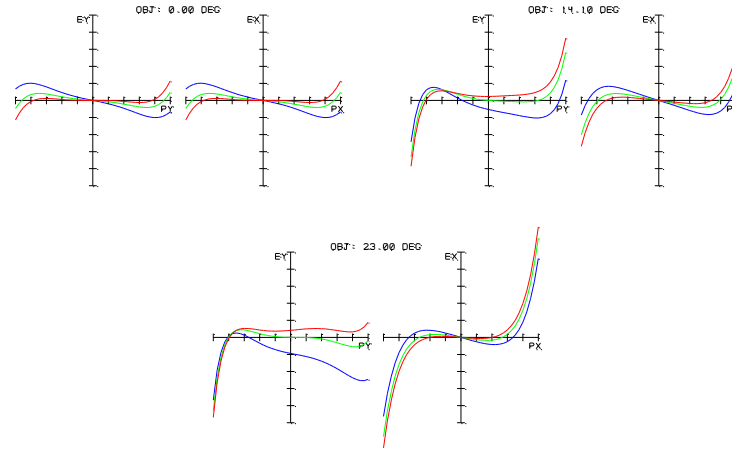
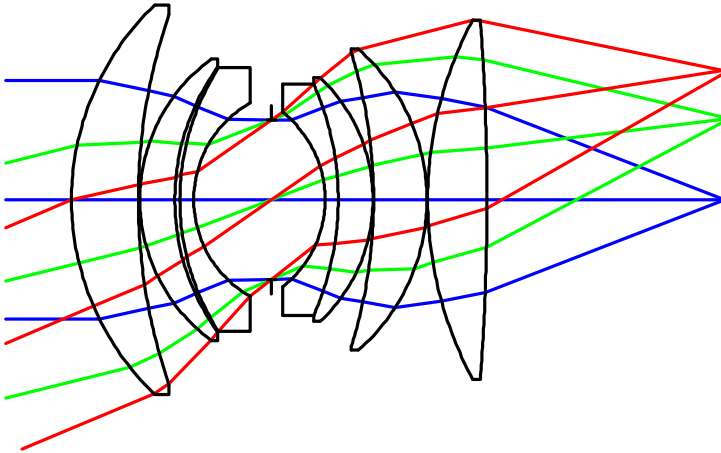
- Imaging of a bar pattern with a coma of 0.4λ in x and y
- Structure size near the diffraction limit
- Asymmetry due to coma seen in comparison of edge slopes



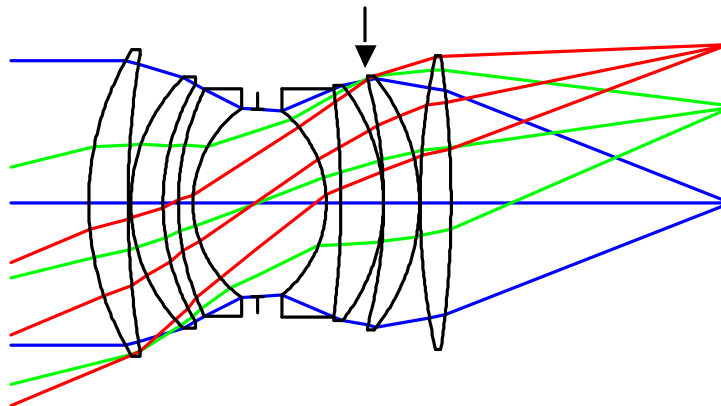


Coma Truncation by Vignetting

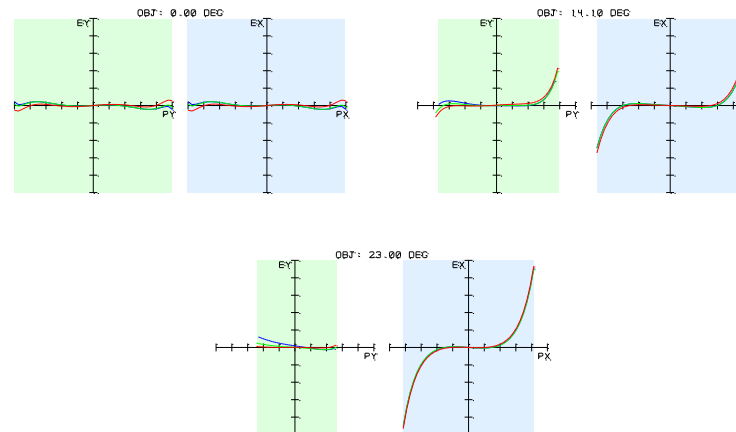
without vignettierung



with vignettierung



tangential / sagittal



Distortion Example: 10%



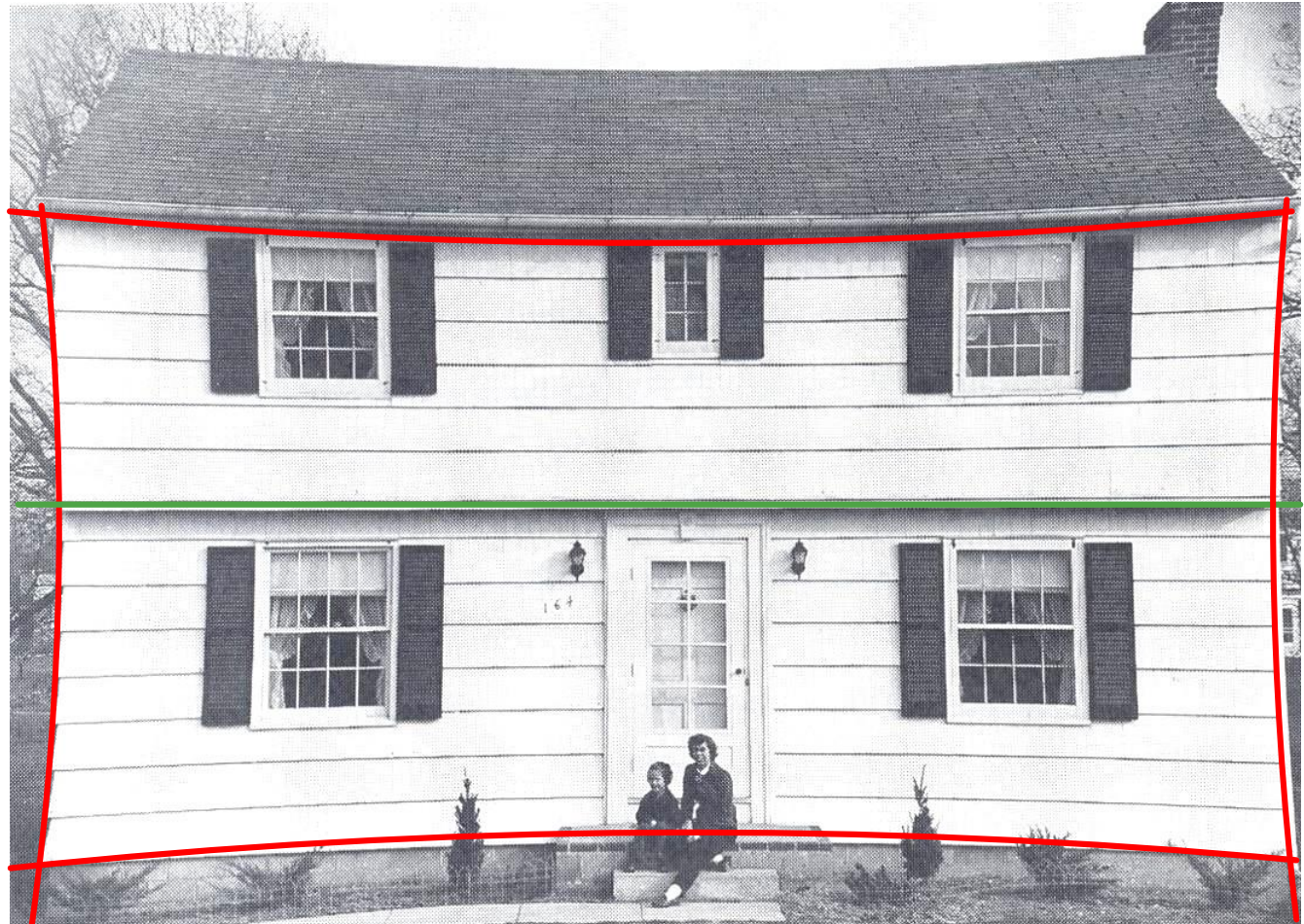
- What is the type of degradation of this image ?
- Sharpness good everywhere !



Ref : H. Zügge

Distortion Example: 10%

- Image with sharp but bended edges/lines
- No distortion along central directions

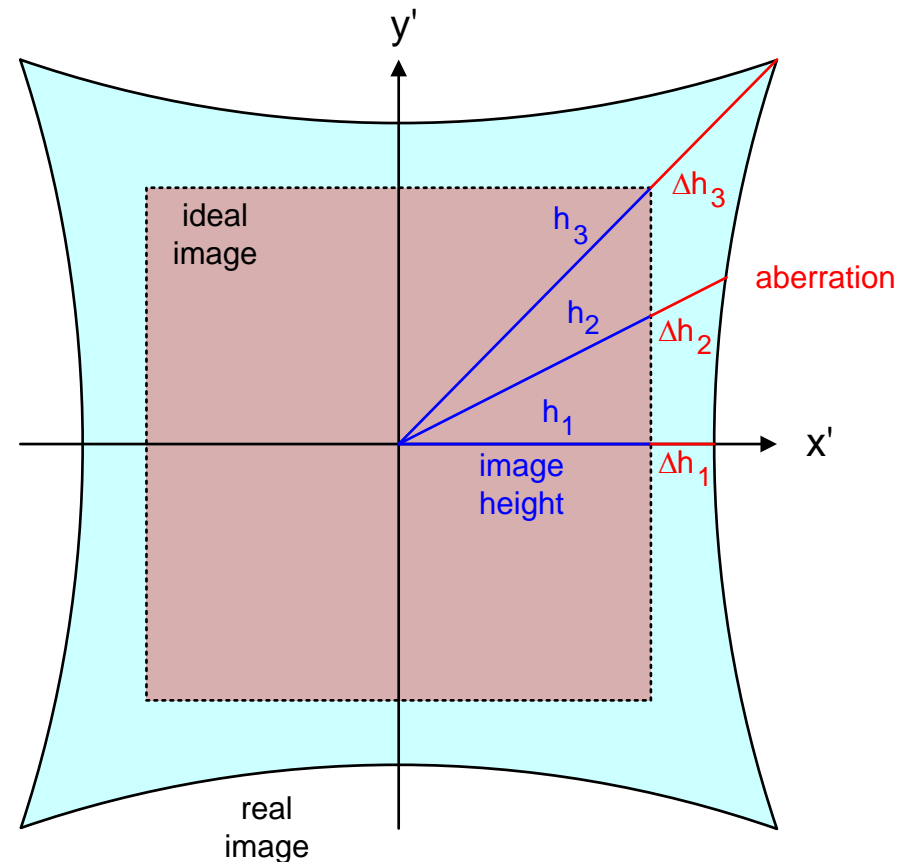


Ref: H. Zügge

- Distortion. change of magnification over the field
- Corresponds to spherical aberration of the chief ray
- Measurement: relative change of image height

$$V = \frac{y_{real} - y_{ideal}}{y_{ideal}}$$

- No image point blurr
only geometrical shape deviation
- Sign of distortion:
 1. $V < 0$: barrel,
lens with stop in front
 2. $V > 0$: pincushion,
lens with rear stop



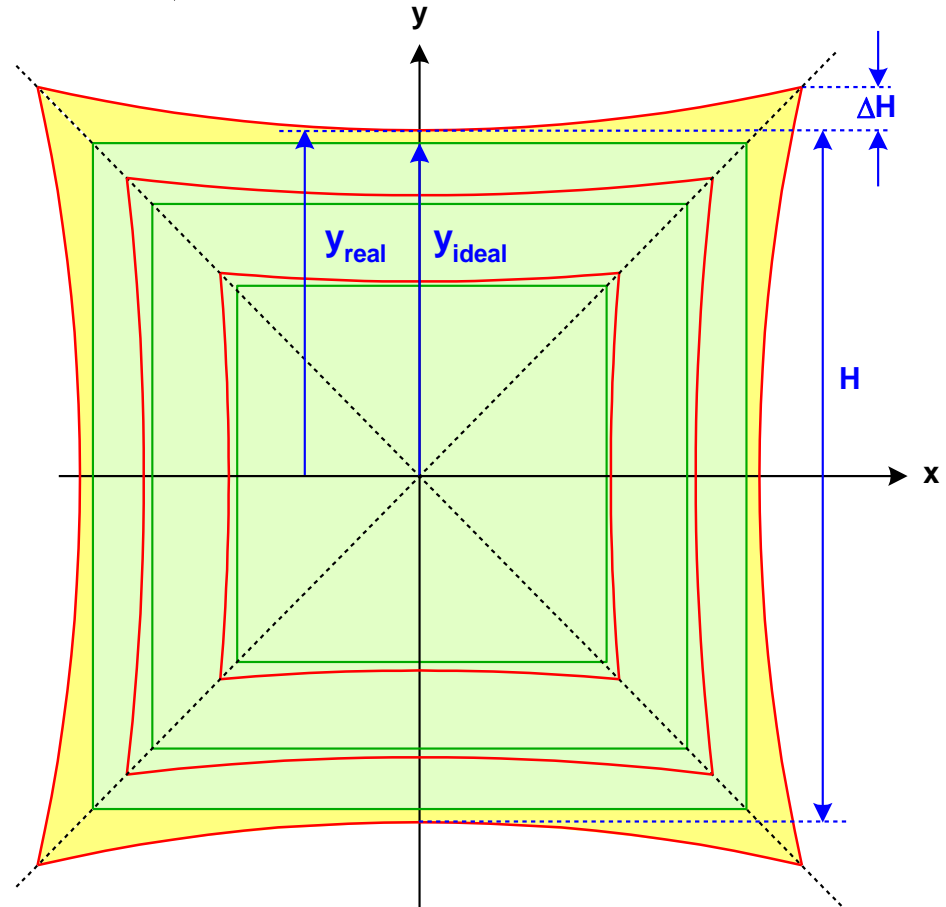
- Conventional definition of distortion

$$V = \frac{\Delta y}{y}$$

- Special definition of TV distortion

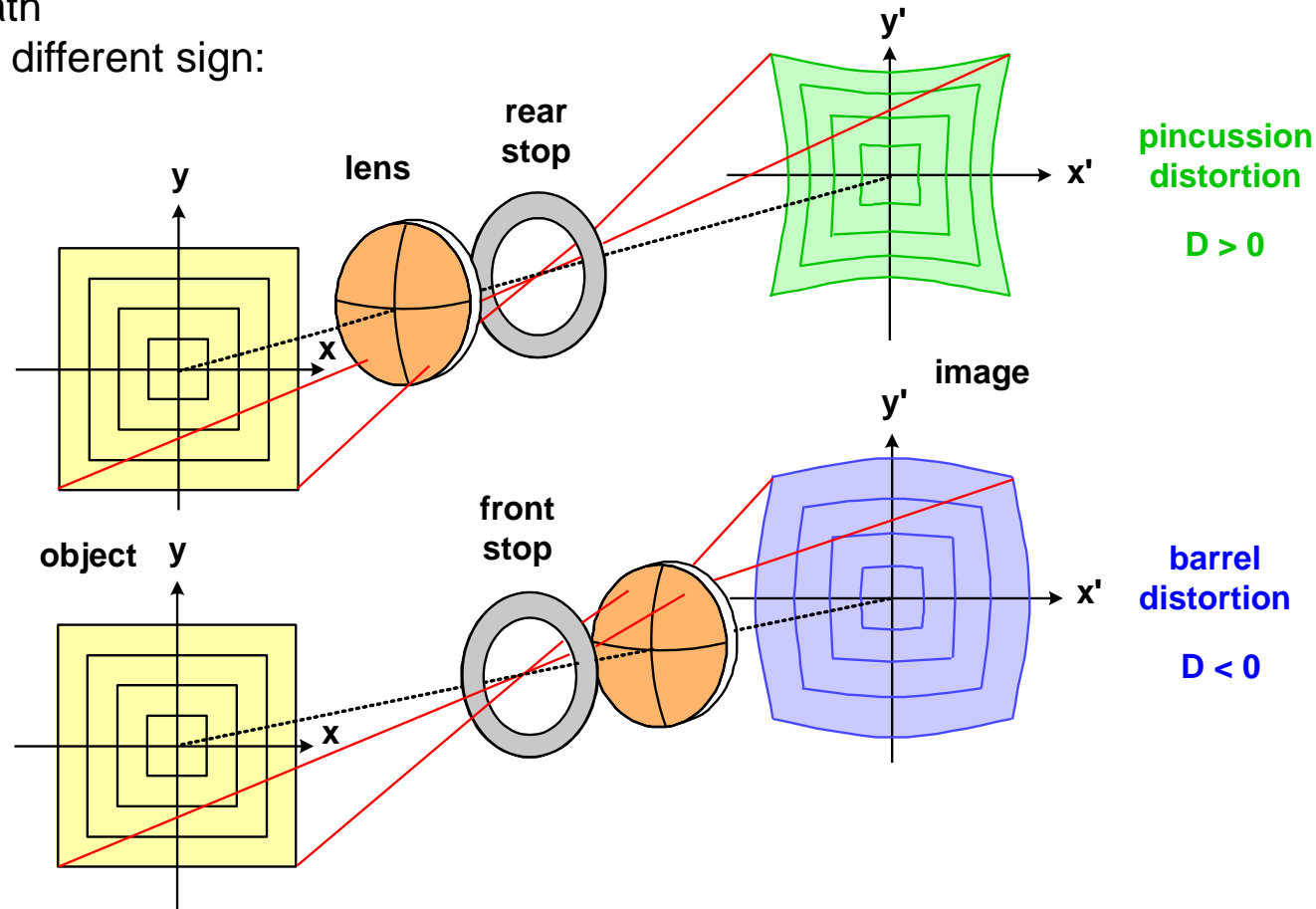
$$V_{TV} = \frac{\Delta H}{H}$$

- Measure of bending of lines
- Acceptance level strongly depends on kind of objects:
 - geometrical bars/lines: 1% is still critical
 - biological samples: 10% is not a problem
- Digital detection with image post processing: un-distorted image can be reconstructed



- Purely geometrical deviations without any blurr
- Distortion corresponds to spherical aberration of the chief ray
- Important is the location of the stop:
defines the chief ray path
- Two primary types with different sign:
 1. barrel, $D < 0$
front stop
 2. pincushion, $D > 0$
rear stop
- Definition of local magnification changes

$$D = \frac{y'_{real} - y'_{ideal}}{y'_{ideal}}$$



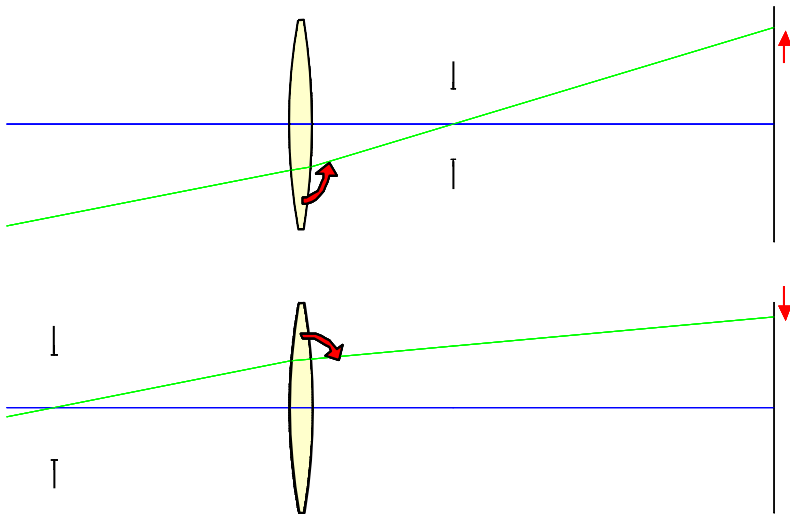


Distortion and Stop Position

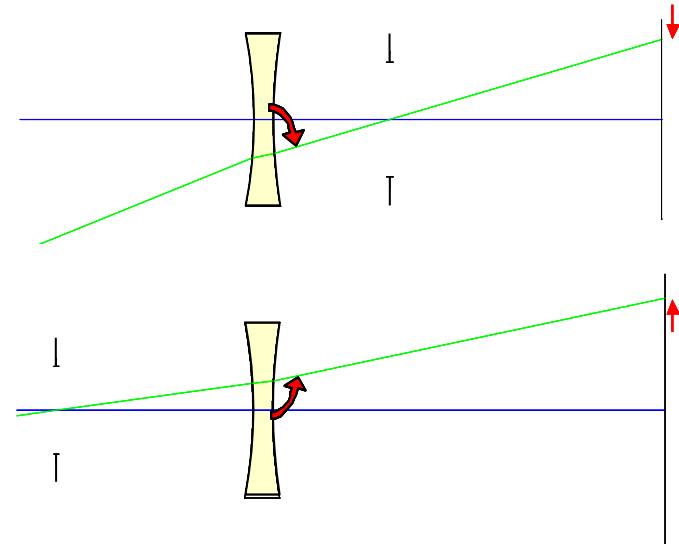
- Sign of distortion of a single lens depends on stop position
- Ray bending of chief ray determines the distortion

Lens	Stop	Distortion	Example
positive lens	rear stop	$D > 0$	Tele lens Loupe
negative lens	front stop	$D > 0$	
positive lens	front stop	$D < 0$	Retro focus lens reversed Binocular
negative lens	rear stop	$D < 0$	

Positive, pincushion

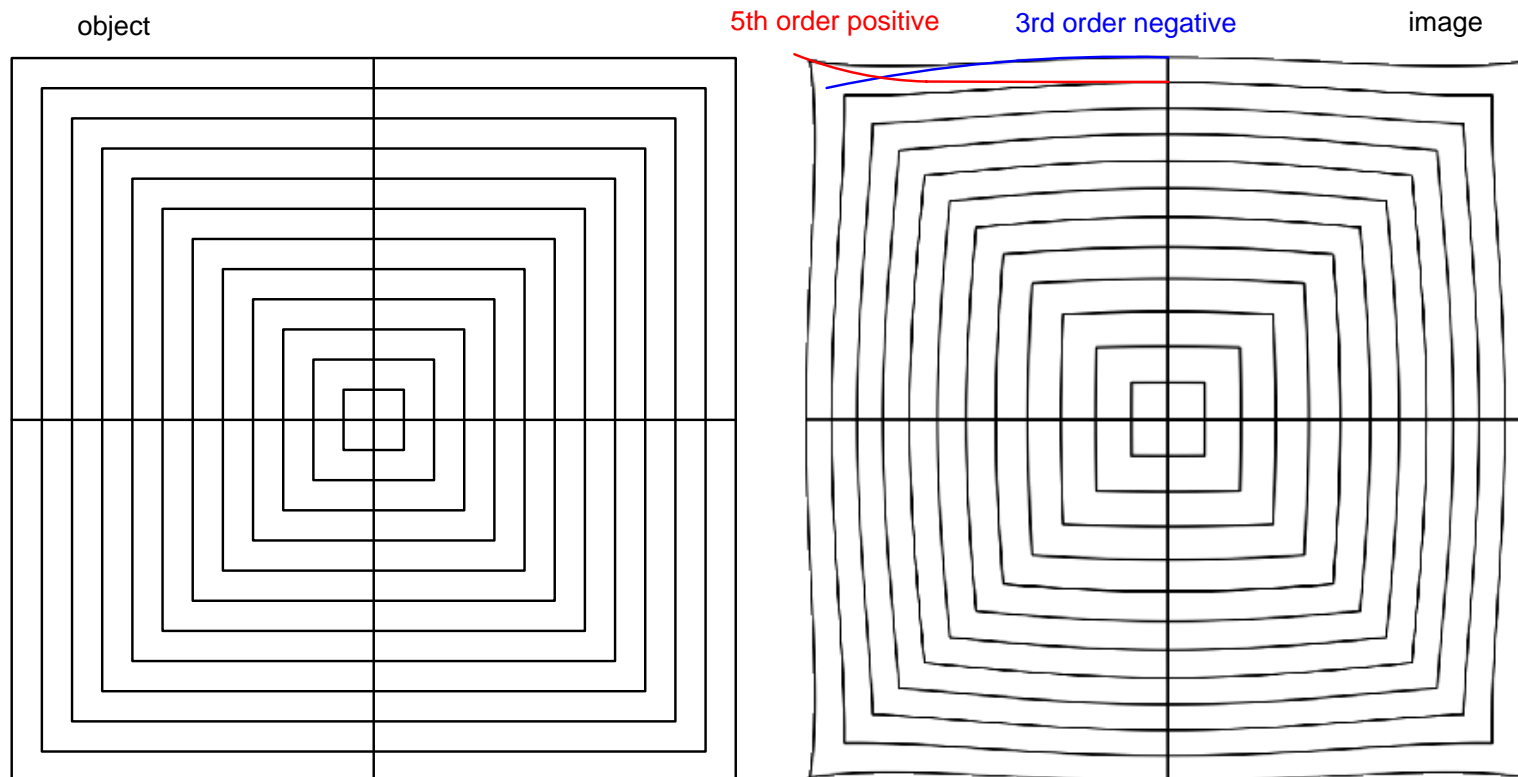


Negative, barrel

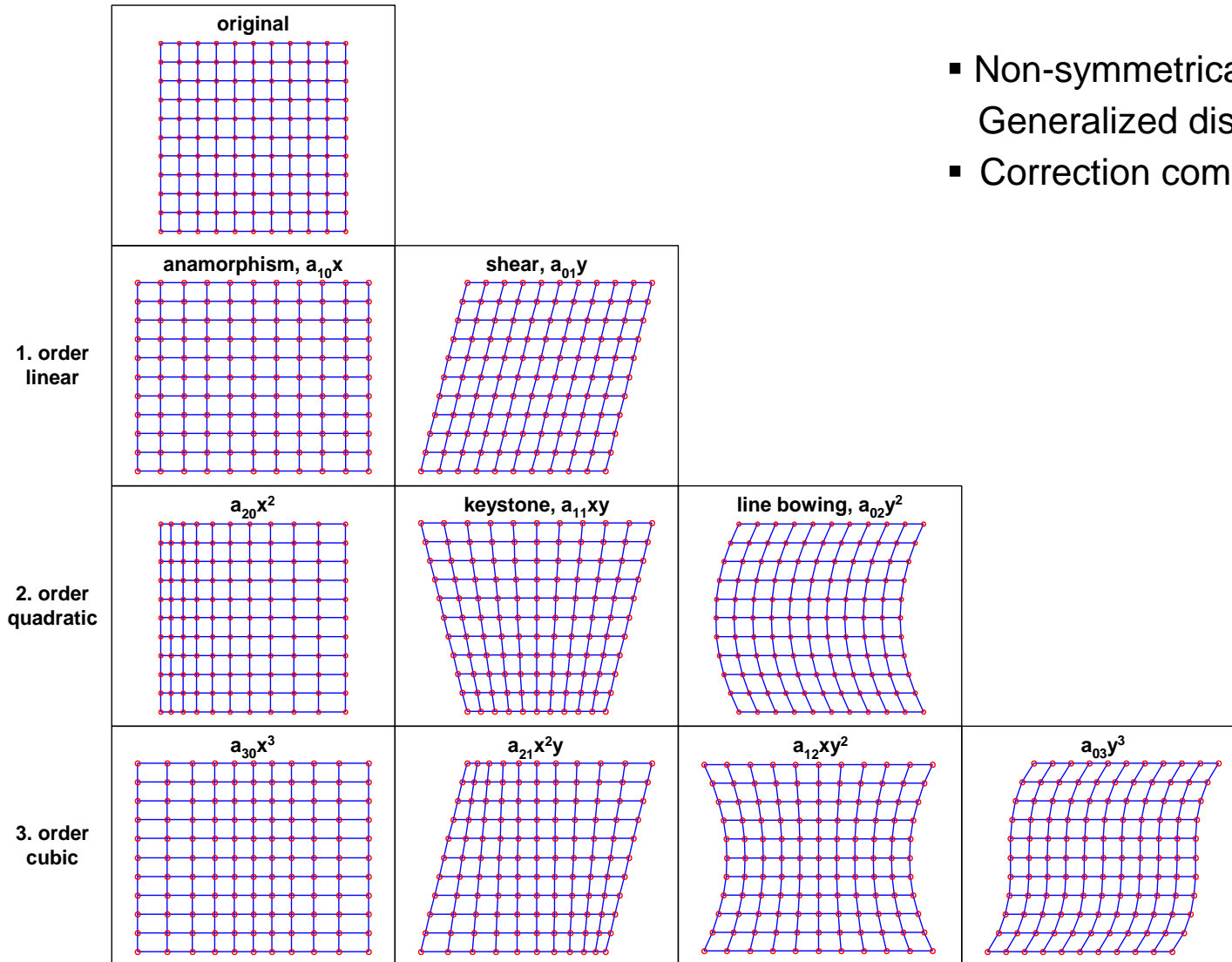


Distortion of Higher Order

- Combination of distortion of 3rd and 5th order:
- Bended lines with turning points
- Typical result for corrected/compensated distortion



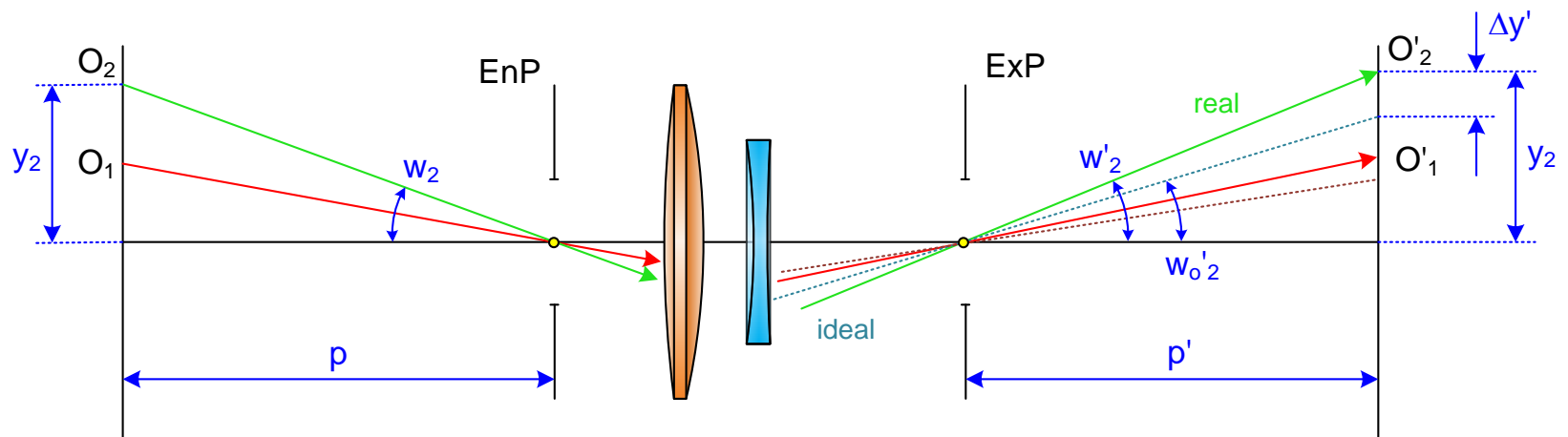
- Non-symmetrical systems:
Generalized distortion types
- Correction complicated



Reasons of Distortion

- Distortion occurs, if the magnification depends on the field height y
- In the special case of an invariant location p' of the exit pupil:
the tangent of the angle of the chief ray should be scaled linear
- Airy tangent condition:
necessary but not sufficient condition for distortion correction:
- This corresponds to a corrected angle of the pupil imaging
from entrance to exit pupil

$$\frac{\tan w'}{\tan w} = \text{const}$$



Reasons of Distortion

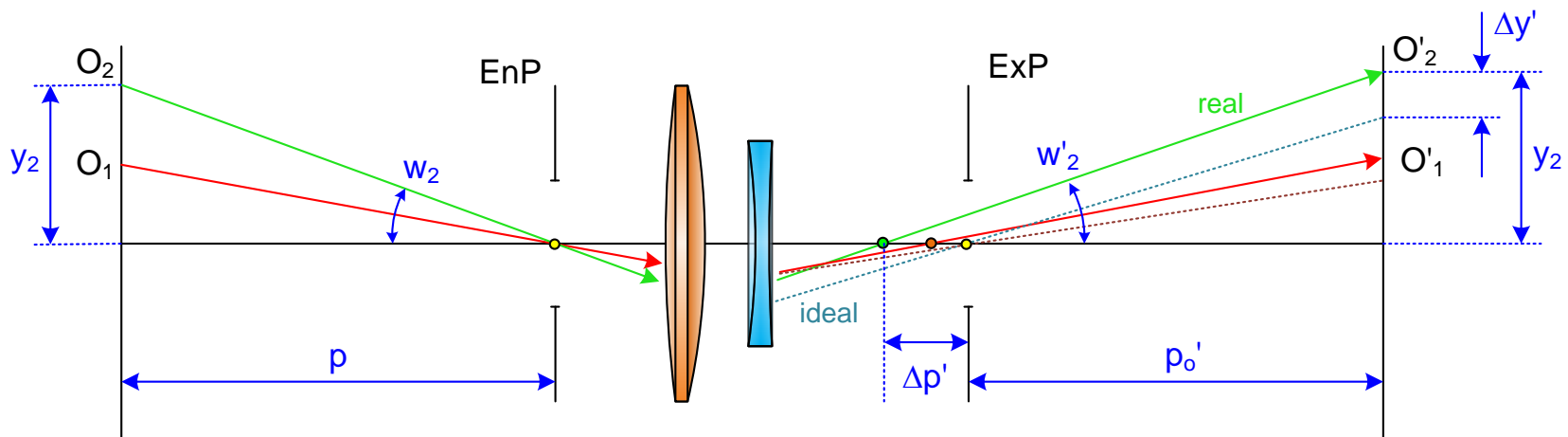
- Second possibility of distortion:
the pupil imaging suffers from longitudinal spherical aberration
- The location of the exit pupil than depends on the field height
- With the simple relations

$$y = p \cdot \tan w \quad , \quad y' = p' \cdot \tan w'$$

we have the general expression
for the magnification

$$m(y) = \frac{y'}{y} = \frac{p'(y) \cdot \tan w'}{p \cdot \tan w} = \left(\frac{p_o'}{p} + \frac{\Delta p'(y)}{p} \right) \cdot \frac{\tan w'}{\tan w}$$

- For vanishing distortion:
 1. the tan-condition is fulfilled (chief ray angle)
 2. the spherical aberration of the pupil imaging is corrected (chief ray intersection point)

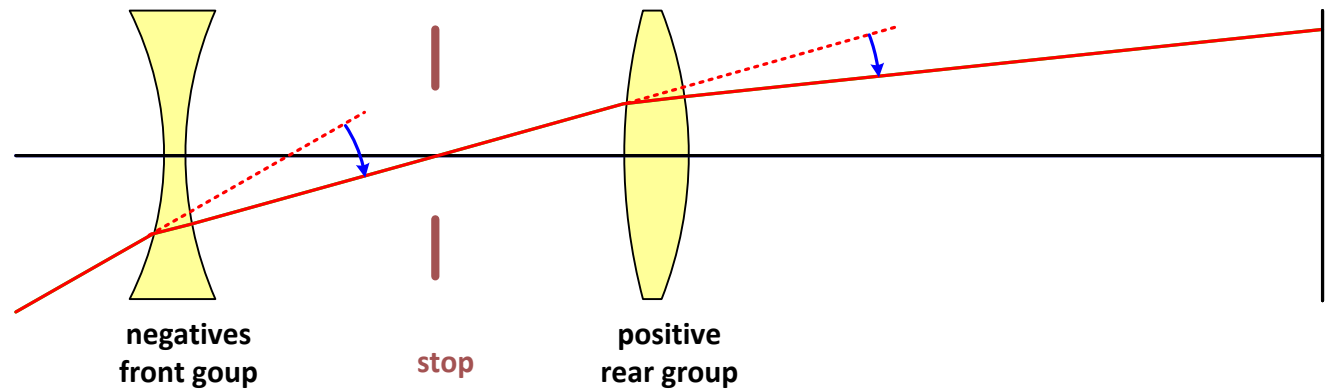
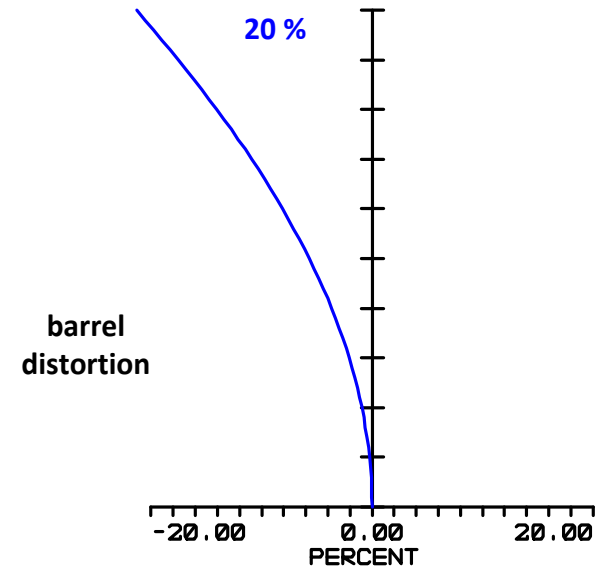
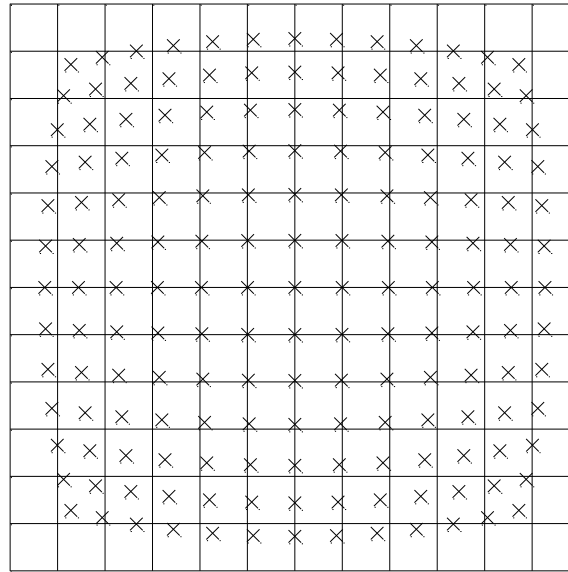




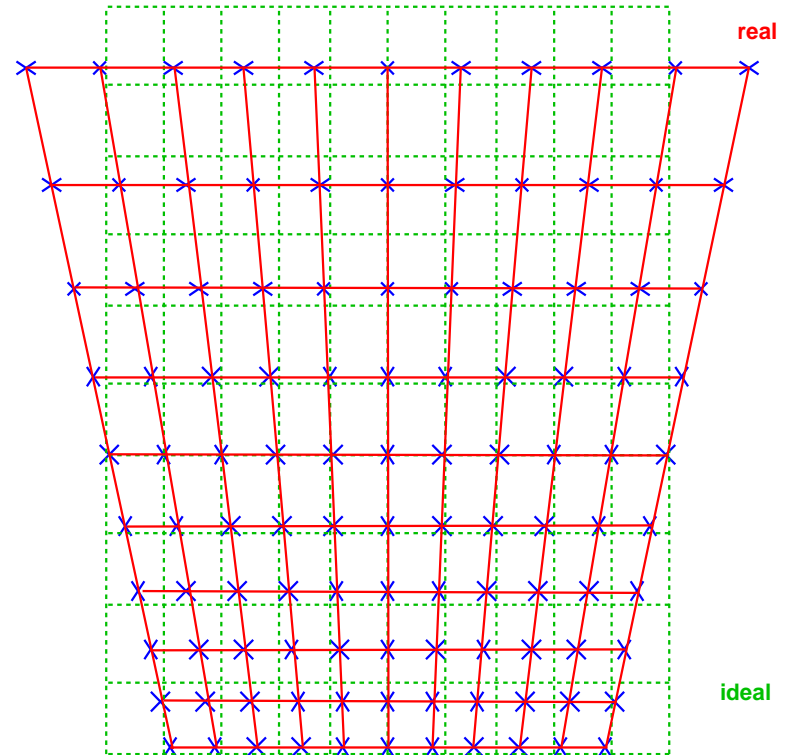
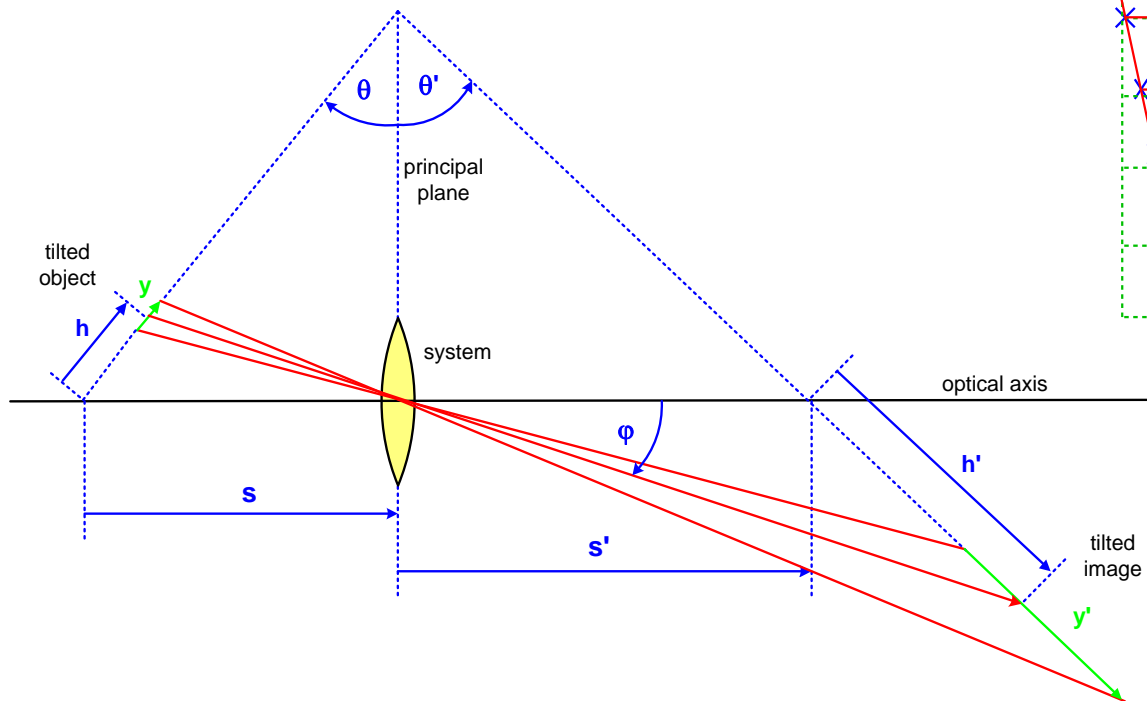
Distortion of a Retrofocus System

Retro focus-
systems :

barrel distortion



Keystone distortion



- Visual impression of distortion on real images
- Visibility only at straight edges
- Edge through the center are not affected

original



20% keystone



barrel and pincushion



5% barrel



10% barrel



15% barrel



2% pincushion



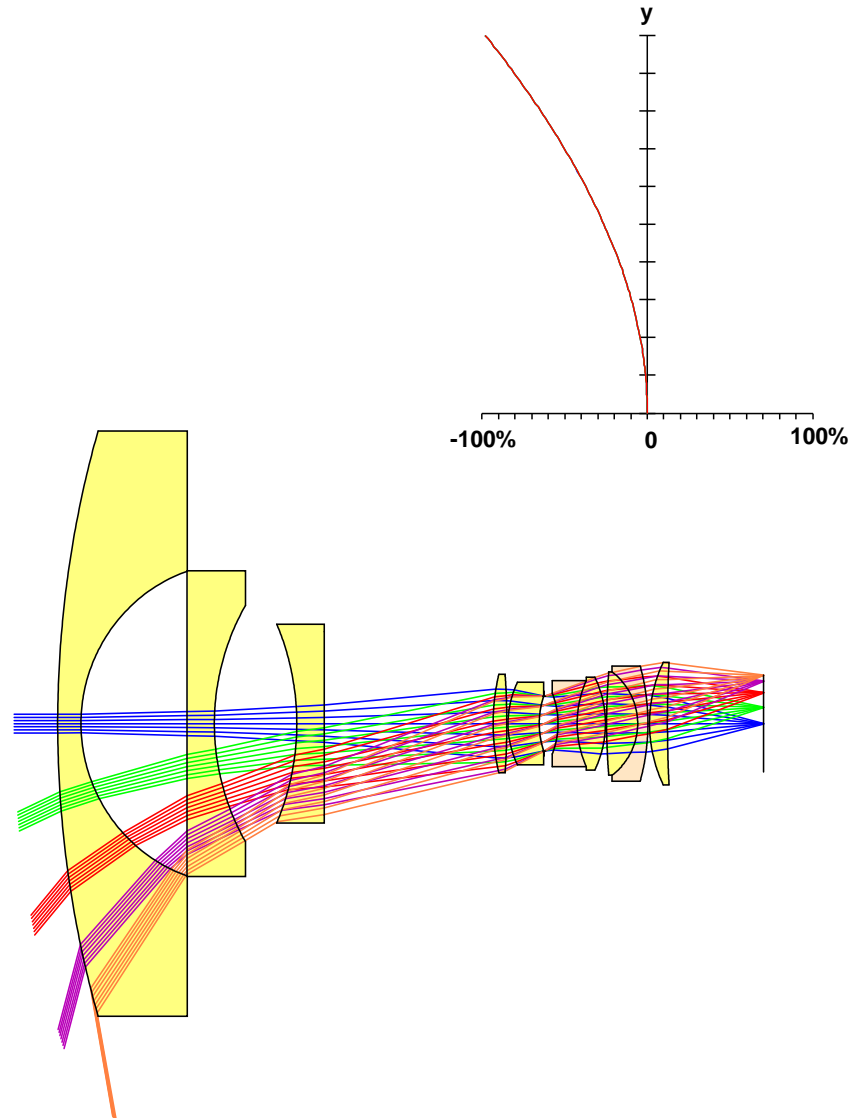
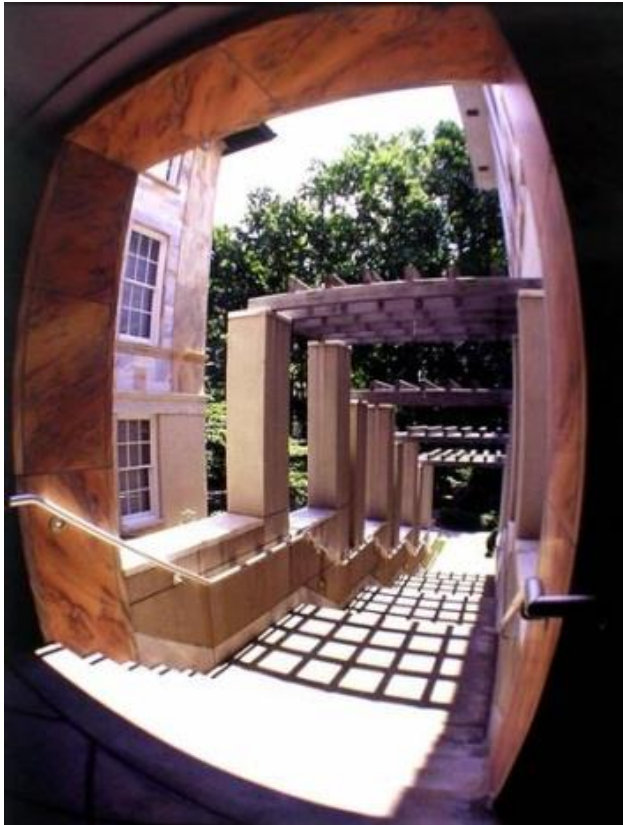
5% pincushion



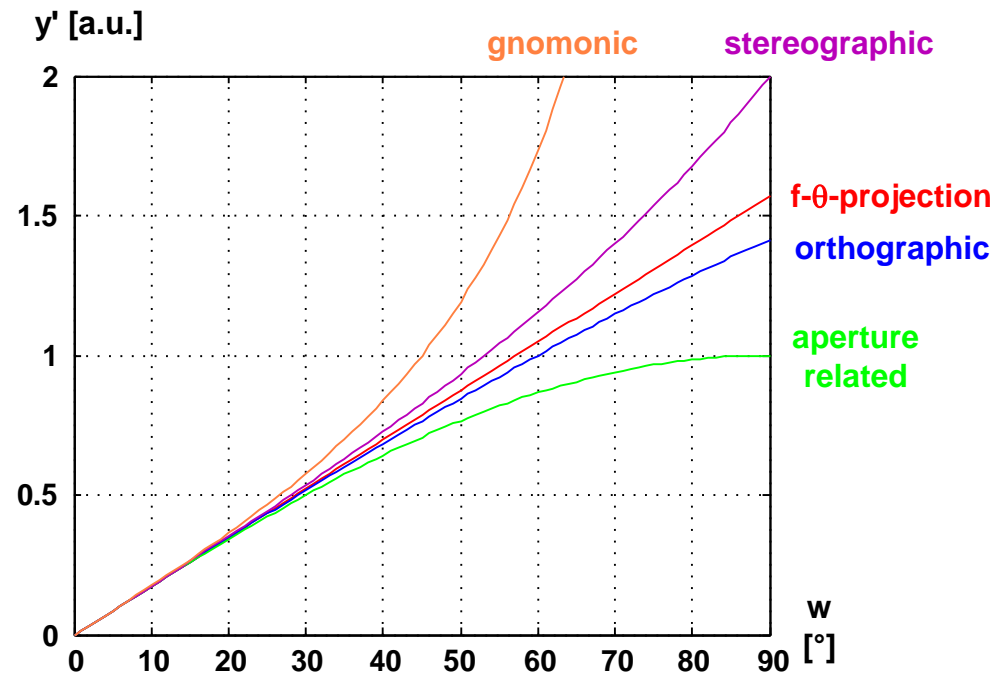
10% pincushion



- Example lens with 210° field of view

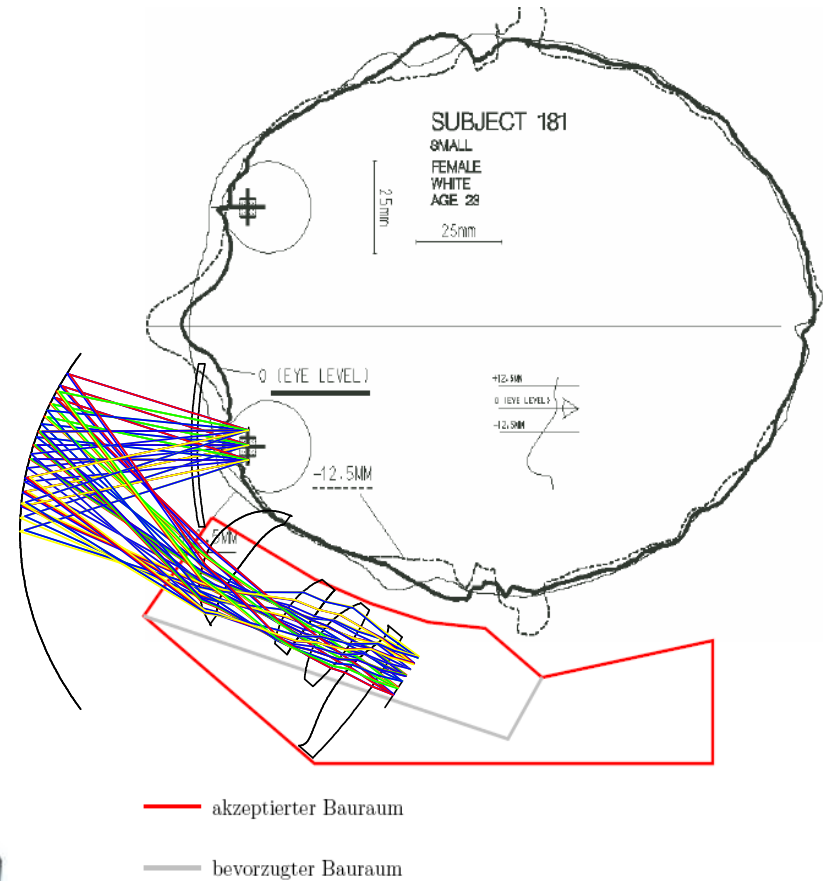


- Distortion types

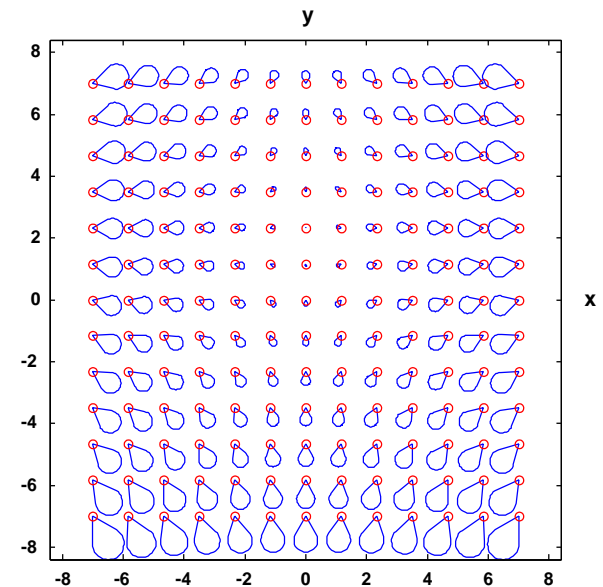
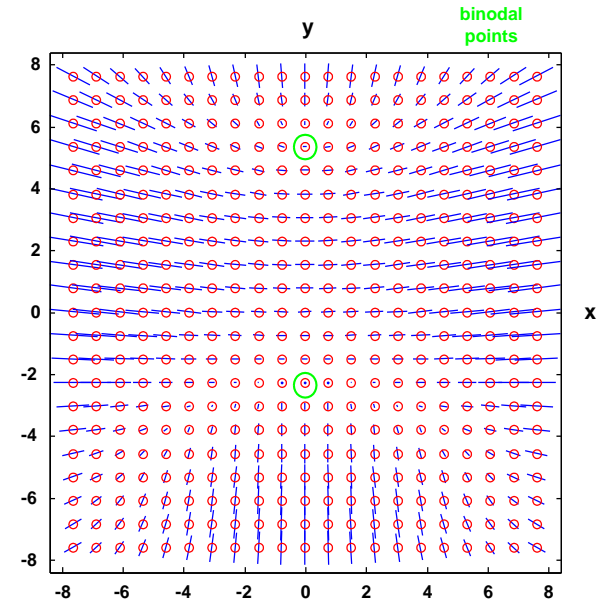
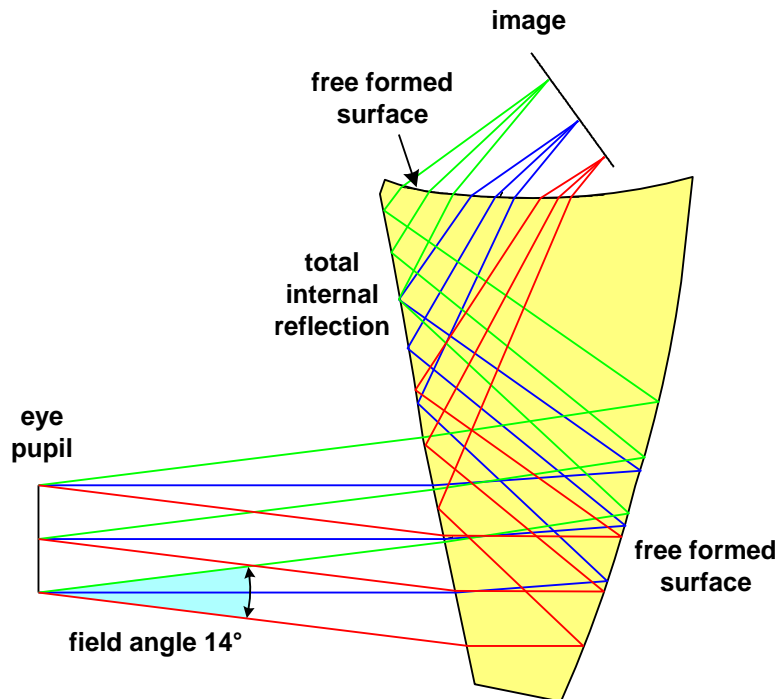


Head Mounted Display

- Commercial system:
Zeiss Cinemizer
- Critical performance of distortion due to asymmetry



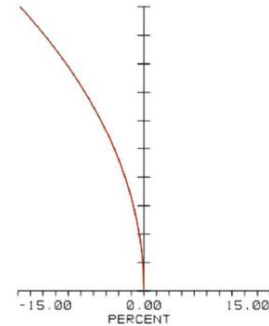
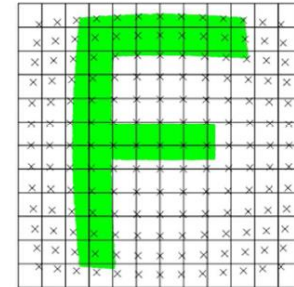
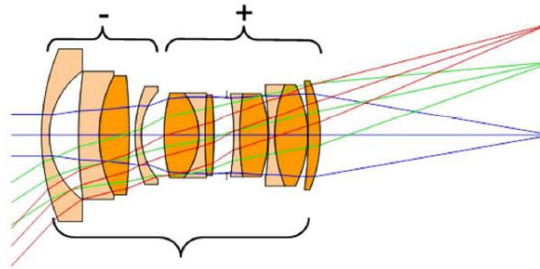
- Refractive 3D-system
- Free-formed prism
- Field dependence of coma, distortion and astigmatism
- One coma nodal point
- Two astigmatism nodal points



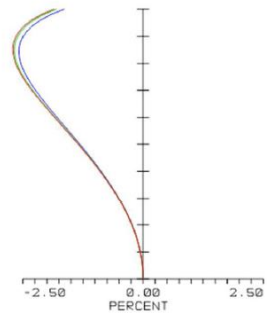
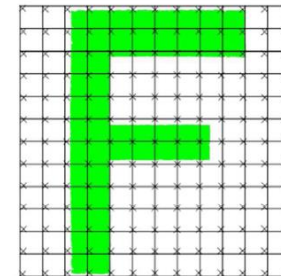
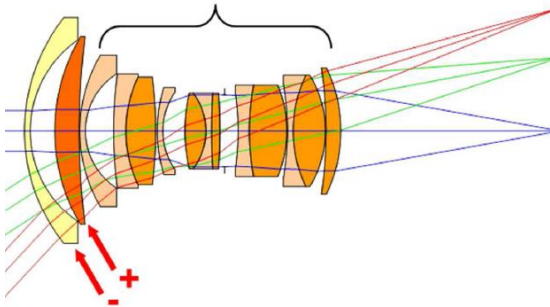
Distortion Correction

- Correction the distortion of a retro focus type photographic lens by additional splitted lens

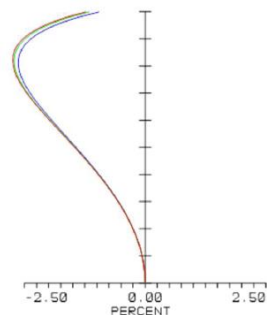
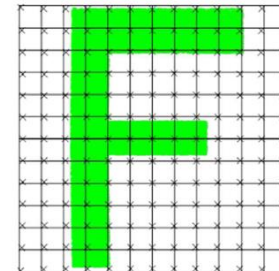
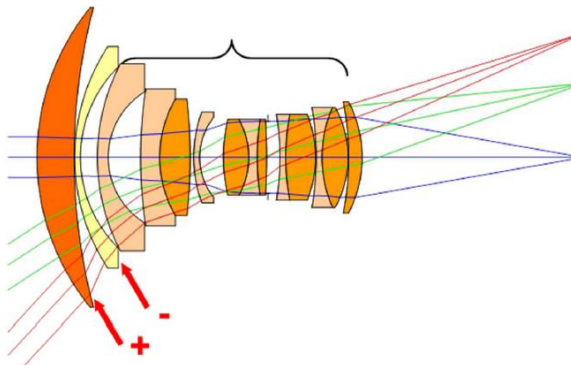
a) distortion not corrected: 15%



b) distortion corrected by - + :2.5%



c) distortion corrected by + - :2.5%





Distortion in Spectrometer

- Spatial distortion, keystone: bendet slit
- Spectral distortion, smile

