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Imaging and Aberration Theory – Seminar 2

Exercise 2-1: Fermat Principle

a) Derive the law of refraction in two dimensions by the Fermat principle of smallest optical path length between two points above and below a plane interface.

b) Assume an aspherical surface between two media with indices n and n' respectively. Compute the exact surface equation under the assumption that a collimated incoming ray bundle is focussed perfectly. Discuss the result for n > n' and n < n'.

Exercise 2-2: Malus-Dupin - Relation between Rays and Waves

a) Show that rays and the corresponding wavefronts (surfaces of constant optical path length) starting from the same object point remain orthogonal to each other during the propagation through an optical system.

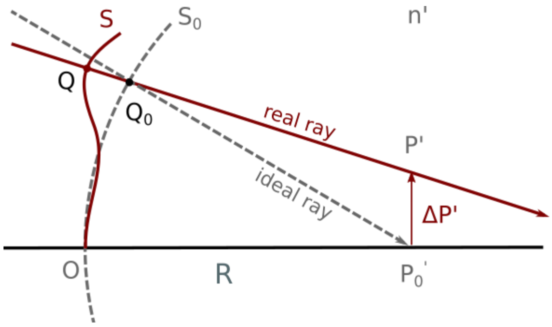
b) Consider an optical system imaging an axial object point P to the image point P_0'. If P and P_0' are *perfect conjugates*, the outgoing wavefront S_0 at the exit pupil is a perfect sphere centered on P_0'. If aberrations are present, the real wavefront S deviates from this ideal sphere. A real ray, which passes through the exit pupil point $Q_0 = (x_p, y_p, z_p)$, no longer intersects the image plane in the ideal image point P_0', but in a point P'. The difference $\Delta P' = (\Delta x, \Delta y)$ is the *transverse ray aberration*. The corresponding *wavefront aberration* is defined as the optical path difference between S and S_0 measured along the real ray:

$$W(x_p, y_p) = [PQ] - [PQ_0] = [QQ_0].$$

Prove that transverse ray aberrations are related to wave aberrations by

$$\Delta x \approx -\frac{R}{n'}\frac{d}{dx_p}W(x_p, y_p),$$

$$\Delta y \approx -\frac{R}{n'}\frac{d}{dy_p}W(x_p, y_p).$$



Exercise 2-3: Eikonal – Gradient Index Profile

Consider the Eikonal equation in the parameterized form

$$\frac{\partial}{\partial s}\left(n\frac{d\vec{r}}{ds}\right)=\vec{\nabla}n\,, \tag{1}$$

where ds is the length of the curved line element, n is the refractive index and *r* is the "coordinate of the ray". Transform the equation to the paraxial case (ds$\approx$dz) and solve the result for a two-dimensional ray curve parameterization x=x(z), y=0. Assume a parabolic index distribution of the form

$$n^2(x)=n_0^2\left(1-\alpha x^2\right), \tag{2}$$

where α is a positive and real constant and is assumed to be small..